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Information Content of Inflation Expectations: A Copula-Based Model

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Abstract: This paper introduces a holistic framework that integrates copula modeling and information-theoretic measures to examine the information content of inflation expectations. Copulas are used to capture the dynamic dependence between inflation and expectations, accounting for extreme events and tail dependence. Information-theoretic measures are employed to quantify the information that expectations provide about inflation. Theoretical results establish a link between copula entropy and mutual information, propose a lower bound for copula entropy, and provide a practical tool for central banks to anchor expectations to achieve inflation targets. Empirical findings reveal higher uncertainty in the tails of the joint distribution and underscore the meaningful information carried by expected inflation for forecasting inflation, particularly with shorter-term expectations.

Keywords: copula modeling; dependence structure; entropy; Fisher information; inflation expectations; mutual information

JEL Classification: C15; C40; D84; E31; E52

1 Introduction

The relationship between inflation and expectations is of utmost importance for central banks, as expectations directly influence the behavior of market participants in setting prices and wages. This paper presents a hybrid framework integrating copulas for modeling dependencies and information-theoretic measures for quantifying uncertainty to address dynamics between inflation and expectations. This paper builds upon existing literature on copula and entropy methods (Ariel and Louzoun 2020; Ma and Sun 2011; Sun et al. 2019; Zhao and Lin 2011), as well as studies on inflation and inflation expectations (Cavallo, Cruces, and Perez-Truglia 2017; Coibion, Gorodnichenko, and Ropele 2020; Haubrich, Pennacchi, and Ritchken 2012; Madeira and Zafar 2015) to provide a holistic approach to capturing the dynamics of inflation and expectations.

The literature highlights the role of expectations in shaping inflation dynamics. Chan, Clark, and Koop (2018) develop a bivariate model to examine the relationship between inflation and long-run inflation expectations. According to Cavallo, Cruces, and Perez-Truglia (2017), inflation expectations can be an essential driver of inflation, as they influence the behavior of households, firms, and policymakers. Coibion, Gorodnichenko, and Ropele (2020) further emphasize that expectations can impact actual inflation, affecting the formation of wage contracts, price-setting behavior, and monetary policy decisions. Haubrich, Pennacchi, and Ritchken (2012) also argue that central banks often consider inflation expectations as an important factor in their monetary policy decisions, as they provide valuable information about future inflation trends. Furthermore, Madeira and Zafar (2015) highlight that inflation expectations can serve as the key policy tool for central banks in managing inflation, as they can influence the behavior of economic agents and affect inflation outcomes. Clark and Nakata (2008), Kozicki and Tinsley (2012), and Faust and Wright (2013) extensively study the behavior of expected inflation

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in forecasting inflation. These studies underscore the significant relationship between inflation and inflation expectations.

This paper employs copulas and mutual information to study the relationship between inflation and expectations. Copulas are statistical tools commonly used to estimate the dependence structure. They allow for estimating the joint distribution based on marginal distributions, capturing the dependence without making assumptions about marginal distributions (Joe 1997; Nelsen 2006). On the other hand, mutual information quantifies the amount of information shared between inflation and expectations, reflecting the nature of the dependence between the two. Mutual information is a nonparametric measure that does not require assumptions about the underlying distributions (Cover and Thomas 1991), making it a powerful tool for evaluating the strength of the dependence between inflation and inflation expectations. The proposed approach provides a framework for capturing the structural and informational aspects of inflation and expectations interaction by combining copulas for constructing the joint distribution and mutual information for quantifying uncertainty.

Copulas can be connected with information-theoretic measures using the notion of entropy introduced by Shannon (1948). The entropy of copula, known as *copula entropy*, measures the uncertainty in a multivariate distribution. Copula entropy has been widely applied in econometrics and finance to study the risk in various events (Cherubini, Luciano, and Vecchiato 2004; Fang, Kotz, and Ng 1990). The relationship between copula functions and mutual information can also be described using copula entropy. This paper introduces a lower bound for copula entropy, given by the Fisher information (Fisher 1922). The Fisher information quantifies the amount of information that inflation expectations contain about the unknown parameter of the joint distribution of inflation and expectations. The theoretical results illustrate that as the copula entropy of inflation and inflation expectations increase, the uncertainty in their joint distribution also rises, resulting in weaker dependence. This increased uncertainty makes it more challenging for central banks to achieve their inflation targets, indicating that expectations play a crucial role in shaping inflation dynamics. This result provides a computational tool to measure central banks' credibility and monetary policy effectiveness.

The empirical analysis employs data on inflation expectations obtained from the Surveys of Consumers at the University of Michigan, which collect information on the median expected price change over the next 12 months. This data provides insights into how households anticipate prices to change in the near future. Additionally, data on expected inflation rates for 2-, 10-, and 30-year horizons, compiled by the Federal Reserve Bank of Cleveland, and 1- and 10-year inflation forecasts from the Survey of Professional Forecasters are also included in the analysis. These estimates are calculated using various sources, including Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations. The monthly data span from January 1982 to November 2022. The empirical findings suggest higher uncertainty in the tails of the copula densities. The results highlight the informative nature of expected inflation for forecasting inflation, primarily when longer-term expectations are used. The University of Michigan survey and the Federal Reserve Bank of Cleveland's 2-year expectations offer better predictions of current inflation than longer-term expectations. These findings have implications for risk management and policy decisions. Firstly, the identification of heavy-tailed copula densities suggests that tail events related to inflation and inflation expectations may be more uncertain and potentially more volatile, which has implications for risk management strategies. Secondly, copula-based approaches with information-theoretic measures can quantify the uncertainty in the relationship between inflation and inflation expectations.

While the literature has explored various techniques to capture the relationship between inflation and expectations, the proposed approach presents several advantages over conventional methodologies. First, it simultaneously addresses dependency structures and quantifies uncertainty. Traditional methods often study these aspects in isolation, which might not capture the comprehensive nature of the relationship between inflation and expectations. Second, copulas allow for taking into account extreme events and tail dependence. This aspect is important because extreme economic conditions (like hyperinflation or deflation) can have significant policy and market implications. Third, the mutual information measure offers a nonparametric dependence assessment, allowing for a more flexible evaluation not constrained by specific distributional assumptions. This is particularly beneficial in cases where the underlying relationship might be non-linear or exhibit properties not easily captured by traditional parametric approaches. Lastly, by understanding the degree of dependence and the embedded uncertainty, central banks can better anchor expectations and adjust their strategies to meet inflation targets effectively.

The paper is organized as follows. Section 1 overviews the copula families and information-theoretic measures. Section 2 establishes a connection between two fundamental notions of copula theory and information theory to capture the information content of inflation expectations. The section proposes a lower bound for copula entropy. Examples and simulations are presented to clarify the concepts and illustrate the practical applications of information-theoretic measures in analyzing the dependence structure. Section 3 describes data and provides the results of the empirical analysis. Section 4 provides concluding remarks. Proofs, technical details, and robustness checks are provided in the Appendix.

2 Preliminaries

This section discusses fundamental concepts that serve as the basis for the proposed framework. First, it briefly overviews copula modeling used to capture interdependencies. Next, it elaborates on information-theoretic measures employed for quantifying uncertainties and dependencies. By utilizing these principles, we can examine the information contained within inflation expectations.

2.1 Copula Modeling

The dependence between inflation and inflation expectations can be modeled through copulas. Copula functions allow us to model the relationship between different variables and are commonly used in financial modeling and risk management, where the relationship between multiple variables is important to capture (Embrechts et al. 2003). The following properties make copula functions powerful computational tools for modeling the dependence structure. First, a copula function can uniquely determine a multivariate distribution, given known marginal distributions. Copula functions are also symmetric and nonparametric. In addition, copula functions are always in the range [0,1] (Nelsen 2006). To apply copulas, we convert the random component of inflation and inflation expectation (π, π^e) to u_π and u_{π^e} with standard uniform distribution. The copula function joins multivariate distributions to their marginal uniform distribution. A bivariate copula $C\left(u_\pi, u_{\pi^e}\right)$ is a cumulative distribution function (CDF) with uniformly distributed marginals. Using the probability integral transformation, $\left(u_\pi, u_{\pi^e}\right) = (F(u_\pi), F\left(u_{\pi^e}\right))$, where $F(\pi)$ and $F(\pi^e)$ are marginal CDFs (Joe 1997). Sklar (1959) shows for $\left(u_\pi, u_{\pi^e}\right)$ with joint CDF $F(\pi, \pi^e)$, a copula exists such that

$$F(\pi, \pi^e) = C\left(F(\pi), F(\pi^e)\right), \tag{1}$$

mapping the marginal CDFs to the joint CDF. The commonly used copulas are Archimedean and elliptical families. An elliptical copula corresponds to an elliptical distribution by Sklar's theorem (Fang, Kotz, and Ng 1990). The elliptical copula is given by

$$C\left(u_\pi, u_{\pi^e}\right) = F\left[F^{-1}(u_\pi), F^{-1}\left(u_{\pi^e}\right)\right], \tag{2}$$

where F^{-1} is the inverse (quantile) function of marginal CDF. The probability density function (PDF) of a copula is given by differentiating Equation (1) as

$$c\left(F(\pi), F(\pi^e)\right) = \frac{f(\pi, \pi^e)}{f(\pi) f(\pi^e)}, \tag{3}$$

where $f(\pi, \pi^e)$ is the joint PDF, and $f(\pi)$ and $f(\pi^e)$ are marginal PDFs. Hence, the joint PDF can be decomposed into the product of the marginal and copula densities. If π and π^e are independent, $c = 1$ since $f(\pi, \pi^e) = f(\pi) f(\pi^e)$. Given π and π^e dependence, $c \neq 1$. Differentiating Equation (2) gives the PDF of the elliptical copula

$$c\left(u_\pi, u_{\pi^e}\right) = \frac{f\left[F^{-1}(u_\pi), F^{-1}\left(u_{\pi^e}\right)\right]}{f\left[F^{-1}(u_\pi)\right] f\left[F^{-1}\left(u_{\pi^e}\right)\right]}. \tag{4}$$

Ebrahimi, Jalali, and Soofi (2014) provide insights into dependence among the components of a random vector for the elliptical families. The Archimedean copula is constructed by

$$C\left(u_\pi, u_{\pi^e}\right) = \varphi^{-1}\left[\varphi(u_\pi), \varphi\left(u_{\pi^e}\right)\right], \tag{5}$$

where φ is a monotone generator function with its inverse φ^{-1}. Differentiating Equation (5) gives the copula PDF, whose functional form depends on the generator function.

Known copula densities are Gaussian and Student-*t* from the elliptical family and Clayton and Frank from the Archimedean family. Table 1 presents these copula densities. Gaussian copulas are commonly used to model dependence. They are specified by a linear correlation structure ρ. As ρ increases, the probability of their joint occurrence also increases. Student-*t* copulas are similar to Gaussian but allow for heavier tails. This means there is a higher probability of extreme values occurring with a Student-*t* than with a Gaussian copula. This feature makes them useful for modeling situations with a significant risk of extreme events (Genest, Gendron, and Bourdeau-Brien 2009). Like Gaussian copulas, Student-*t* copulas are defined by ρ, but the PDF is identified by a Student-*t* distribution instead. Clayton copulas are characterized by a lower tail dependence, meaning extreme events are more likely to occur together and exhibit a negative dependence structure. Frank copulas are characterized by a flexible dependence structure with a positive dependence structure (Joe 1997).

Figure 1 displays the copula densities described in Table 1, with variations in different parameters. The Gaussian copulas exhibit symmetry and have bell-shaped probability density functions (PDFs). When the variance is small, the density of the Gaussian copula is more concentrated around the center, indicating a higher likelihood of observing values close to the median. As the variance increases, the copula density becomes wider and flatter, indicating a higher probability of observing a wider range of values. On the other hand, the Clayton and Frank copulas have PDFs that are more concentrated in the tails as the parameter increases. This means that as the parameter of the Clayton and Frank copulas increases, the probability of observing extreme events (i.e. values far from the median) occurring together also increases. In other words, the dependence between the variables represented by the Clayton and Frank copulas becomes stronger in the tails, suggesting a higher likelihood of extreme events jointly occurring.

Table 1: Known copula densities.

Gaussian	$\frac{1}{\sqrt{1-\rho^2}} \exp\left(\frac{2\rho\Phi^{-1}(u_\pi)\Phi^{-1}(\pi_e)-\rho^2\{[\Phi^{-1}(\pi)]^2+[\Phi^{-1}(\pi^e)]^2\}}{2(1-\rho^2)}\right)$
Student-*t*	$\frac{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\nu+2}{2}\right)\left(1+\frac{t_\nu^{-2}(u_\pi)+t_\nu^{-2}\left(u_{\pi^e}\right)-2\rho t_\nu^{-1}(u_\pi)t_\nu^{-1}\left(u_{\pi^e}\right)}{\nu(1-\rho^2)}\right)^{-\frac{\nu+2}{2}}}{\sqrt{1-\rho^2}\,\Gamma\left(\frac{\nu+1}{2}\right)^2\left(1+\frac{t_\nu^{-2}(u_\pi)}{\nu}\right)^{-\frac{\nu+1}{2}}\left(1+\frac{t_\nu^{-2}\left(u_{\pi^e}\right)}{\nu}\right)^{-\frac{\nu+1}{2}}}$
Clayton	$(1+\theta)(u_\pi u_{\pi^e})^{(-1-\theta)}\left(u_\pi^{-\theta}+u_{\pi^e}^{-\theta}-1\right)^{-2-1/\theta}$, where $\theta \in [-1, \infty), \theta \neq 0$
Frank	$\frac{\theta e^{\theta\left(u_\pi+u_{\pi^e}\right)}(e^\theta-1)}{\left(e^\theta-e^{\theta u_\pi}-e^{\theta u_{\pi^e}}+e^{\theta\left(u_\pi+u_{\pi^e}\right)}\right)^2}$, where $\theta \in (-\infty, \infty), \theta \neq 0$

Φ^{-1} is the inverse CDF of standard normal; $\Gamma(\cdot)$ is the gamma function; t_ν^{-2} is the square of the inverse CDF of the univariate Student-*t*; *t* is the bivariate Student-*t* CDF with linear correlation coefficient ρ and degrees of freedom ν; and θ is a parameter for the generator function.

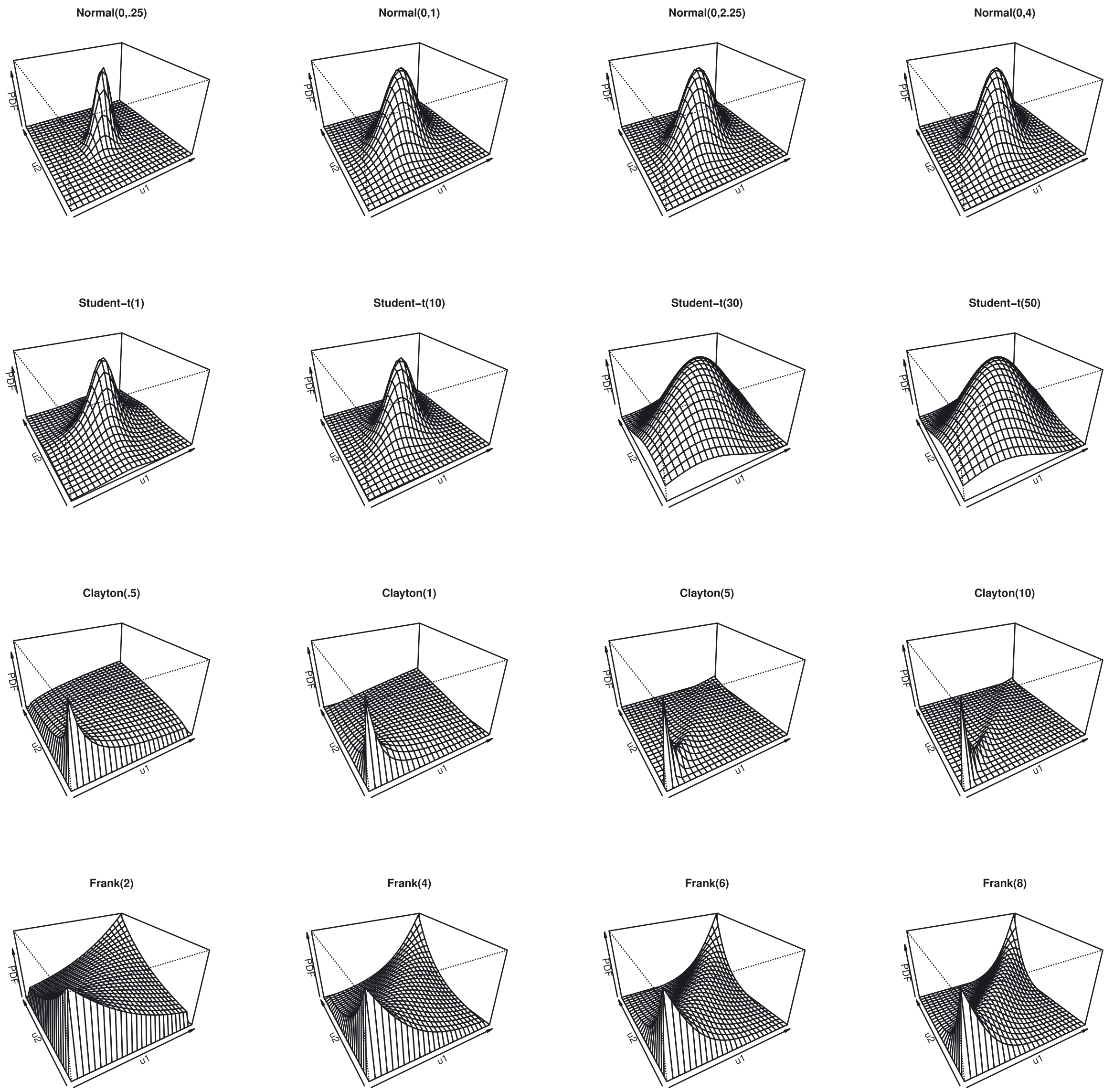

Figure 1: Perspective plots of bivariate Gaussian, Student-*t*, Clayton, and Frank copulas.

2.2 Information-Theoretic Measures

Alternatively, the relationship between inflation and inflation expectations can be studied using mutual information (MI). MI determines the extent to which one random variable holds information about another, thereby indicating the interdependence between the two variables (Lindley 1956). MI is linked to entropy, quantifying the information in a single random variable (Cover and Thomas 1991). Unlike the correlation coefficient, which measures the association between variables, MI is a broader measure that examines the difference between the joint distribution and the product of marginal distributions. MI is often referred to as information gain (Soofi 1994).

Mutual information and copulas are related since both describe the dependence between inflation and inflation expectations. Copulas describe the dependence structure, capturing how inflation and expected inflation depend on each other. On the other hand, MI measures the amount of information inflation contains about inflation expectations and provides a quantitative measure of the dependence between the two. In the previous section, copulas estimate the dependence between inflation and expectations by constructing joint distributions from marginal distributions. In this section, MI is employed to evaluate the strength of the dependence between inflation and inflation expectations. Hence, combining copulas and MI provides a comprehensive approach to studying the information content of inflation expectations.

Given the joint PDF and marginals defined earlier, the mutual information is given by

$$\mathcal{M}(\pi;\pi^e) = \mathbb{E}\left[\log\left(\frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}\right)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e, \tag{6}$$

where $\mathcal{M}(\pi;\pi^e) \geq 0$ and $\mathcal{M}(\pi;\pi^e) = \mathcal{M}(\pi^e;\pi)$. If π and π^e are independent, $\mathcal{M}(\pi;\pi^e) = 0$ where uncertainty is contained in π or π^e independently. This uncertainty is defined by $\mathcal{H}(\pi)$ or $\mathcal{H}(\pi^e)$, where $\mathcal{H}(\cdot)$ denotes Shannon entropy (Shannon 1948) defined as

$$\mathcal{H}(\pi) = -\int_{-\infty}^{\infty} f(\pi)\log(f(\pi))\mathrm{d}\pi. \tag{7}$$

Intuitively, MI captures the information that π and π^e share, measuring how much knowing one reduces uncertainty about the other. The use of entropy and mutual information as tools for understanding the complexities in financial markets and forecasting has been demonstrated in recent studies, such as the work on option pricing by Ardakani (2022a), which incorporates higher-order moments into maximum entropy densities, and the development of ranking forecasts using stochastic error distance and information measures by Ardakani, Ebrahimi, and Soofi (2018). These approaches underscore the value of entropy-based measures in capturing uncertainty of economic data and expectations.

If π and π^e are independent, then knowing π^e does not provide any information about π and vice versa, so $\mathcal{M}(\pi;\pi^e) = 0$. Mutual information can also be expressed by entropies. Let $\mathcal{H}(\pi,\pi^e)$ be joint entropy defined by

$$\mathcal{H}(\pi,\pi^e) = \mathbb{E}\left[-\log\left(f(\pi,\pi^e)\right)\right] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(f(\pi,\pi^e)\right)\mathrm{d}\pi\,\mathrm{d}\pi^e, \tag{8}$$

and $\mathcal{H}(\pi|\pi^e)$ be conditional entropy given by

$$\mathcal{H}(\pi|\pi^e) = \mathbb{E}\left[-\log\left(\frac{f(\pi,\pi^e)}{f(\pi)}\right)\right] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e. \tag{9}$$

$H(\pi|\pi^e)$ provides the uncertainty remaining about π after π^e is known. In other words, it measures what π^e does not say about π. Mutual information can be expressed by conditional and joint entropy as

$$\begin{aligned}\mathcal{M}(\pi;\pi^e) &\equiv \mathcal{H}(\pi) - \mathcal{H}(\pi|\pi^e)\\ &\equiv \mathcal{H}(\pi^e) - \mathcal{H}(\pi^e|\pi)\\ &\equiv \mathcal{H}(\pi) + \mathcal{H}(\pi^e) - \mathcal{H}(\pi,\pi^e).\end{aligned} \tag{10}$$

The proof is provided in Appendix A.1. This formulation of MI represents the uncertainty in π, which is removed by knowing π^e. $\mathcal{M}(\pi^e;\pi^e) \geq \mathcal{M}(\pi;\pi^e)$ since $\mathcal{M}(\pi^e;\pi^e) = \mathcal{H}(\pi^e)$. Also, because $\mathcal{M}(\pi;\pi^e)$ is nonnegarive, $\mathcal{H}(\pi) \geq \mathcal{H}(\pi|\pi^e)$.

MI can also be represented by the Kullback-Leibler divergence (KLD). KLD, or relative entropy, measures the difference between inflation and expected inflation PDFs. It is used to quantify the dissimilarity between the two PDFs. Intuitively, KLD measures how much information is lost when approximating inflation density with expected inflation distribution. The Kullback-Leibler divergence (Kullback and Leibler 1951) is given by

$$\mathcal{K}(\pi;\pi^e) = -\int_{-\infty}^{\infty} f(\pi)\log\left(\frac{f(\pi)}{f(\pi^e)}\right)\mathrm{d}\pi. \tag{11}$$

This measure shows how $f(\pi)$ is different from $f(\pi^e)$. That is the expected excess surprise from using $f(\pi^e)$ as a model when the actual distribution is $f(\pi)$. KLD is non-symmetric and non-negative; it is zero if and only if the two distributions are equal.

MI can be expressed by the Kullback-Leibler divergence from the product of the marginals to the joint distribution. That is,

$$\mathcal{M}(\pi;\pi^e) = \mathcal{K}\big(f(\pi,\pi^e); f(\pi)f(\pi^e)\big) = \mathbb{E}_{\pi^e}\big[\mathcal{K}\big(f(\pi|\pi^e); f(\pi)\big)\big]. \tag{12}$$

The proof is given in Appendix A.2. The properties of these information measures can be found in Ebrahimi, Soofi, and Soyer (2010).

3 Mutual Information and Copula Entropy

This section provides measures that capture the mutual dynamics between inflation and its expectations. Mutual information provides the amount of information shared between the two. On the other hand, copula entropy provides a framework to examine the uncertainty in their joint behavior beyond their marginal distributions. This section first discusses copula entropy and then gives its link with mutual information. This provides a tool for central banks to anchor inflation expectations and achieve inflation targets.

Definition 1. *Copula Entropy (CE) of Inflation and Inflation Expectations: Let $c(u_\pi, u_{\pi^e})$ be a bivariate copula density that captures the dependence structure between inflation π and expectations π^e. The copula entropy is defined as*

$$\mathcal{H}_c(\pi,\pi^e) = -\iint_{[0,1]^2} c\big(u_\pi, u_\pi^e\big)\log\, c\big(u_\pi, u_\pi^e\big)\mathrm{d}u_\pi u_{\pi^e}. \tag{13}$$

The copula entropy measures the uncertainty in the joint distribution of π and π^e that is not captured by their marginal distributions. It provides a measure of the degree of dependence between inflation and expectations that is not explained by their individual variability.

Ma and Sun (2011) show that the MI is a type of entropy leading to the following lemma. This result is derived from Definition 1 and provides a tool for analyzing the joint dynamics of inflation and inflation expectations.

Lemma 1. *Mutual Information and Copula Entropy: The mutual information between π and π^e is equivalent to the negative copula entropy of π and π^e:*

$$\mathcal{M}(\pi;\pi^e) = -\mathcal{H}_c(\pi,\pi^e). \tag{14}$$

The proof is provided in Appendix A.3. Figure 2 depicts the relationship between mutual information, entropy, and copula entropy, along with the dependencies required for computation. It illustrates that the mutual information between inflation and inflation expectations equals the reduction in entropy of inflation due to the information in expected inflation and is equal to the reduction in entropy of expected inflation due to the information in inflation. The arrows indicate the information flow needed for computing each measure, with dashed arrows representing additional dependencies. For example, blue dashed arrows show that entropies can be computed using joint and marginal probability density functions. Red dashed arrows indicate that the mutual information can be calculated by entropies.

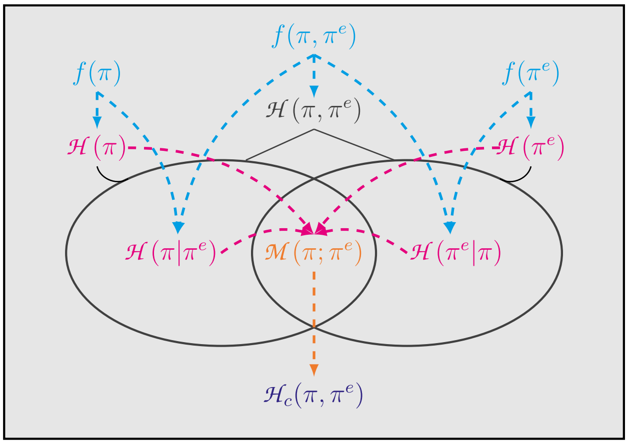

Figure 2: The relationship between mutual information, entropy, and copula entropy. The dashed arrows indicate the information flow needed for computing each measure.

Example 1. *Consider the following example to illustrate how the mutual information between two Gaussian random variables can be calculated using* Equation (10). *The mutual information between two Gaussian random variables can be expressed as* $\mathcal{M}(\pi; \pi^e) = -\frac{1}{2}\log(1-\rho^2)$, *where* ρ *is the correlation coefficient between the two. This result can be obtained from the entropies of the individual variables and their joint entropy. Specifically, for the normal distribution, the entropy of a random variable* π *is given by* $\mathcal{H}(\pi) = \frac{1}{2}\log(2\pi e)\sigma^2$. *Similarly, the joint entropy of two normal random variables* π *and* π^e *is given by* $\mathcal{H}(\pi, \pi^e) = \frac{1}{2}\log[(2\pi e)^2\sigma^4(1-\rho^2)]$. *From* Equation (10), *we can obtain mutual information by*

$$\begin{aligned}\mathcal{M}(\pi; \pi^e) &= \mathcal{H}(\pi) + \mathcal{H}(\pi^e) - \mathcal{H}(\pi, \pi^e)\\ &= \frac{1}{2}\log(2\pi e)\sigma^2 + \frac{1}{2}\log(2\pi e)\sigma^2 - \frac{1}{2}\log[(2\pi e)^2\sigma^4(1-\rho^2)]\\ &= \frac{1}{2}\log\frac{1}{1-\rho^2}\\ &= -\frac{1}{2}\log(1-\rho^2). \end{aligned} \tag{15}$$

Figure 3 *shows three perspective plots of joint PDFs of two simulated Gaussians (n = 10,000). The corresponding MI values are also reported. In the left plot, ρ = 0.1 (weakly correlated). As a result, the joint PDF is elongated. The MI for this case is 0.006, indicating that very little information is shared between the two Gaussians. In the middle plot, ρ = 0.5 (moderately correlated). As a result, the joint PDF appears as a single, wider bump. The MI value for this case is 0.151, indicating a moderate amount of information shared. In the right plot, ρ = 0.9 (strongly correlated). The joint PDF appears as a single, nearly circular bump. The MI value for this case is 0.823, indicating a high amount of information shared between the two Gaussians. The figure illustrates how the amount of information shared depends on their correlation. When the covariance is small, the two variables are largely independent and carry little shared information. When the covariance is large, the two variables are strongly correlated and carry more shared information.*

Example 2. *Consider the following example to illustrate the relationship between correlation, mutual information, and copula entropy. Figure* 4 *plots MI and CE for multivariate Gaussians with* $\mu = 0, 1 \leq \sigma^2 < 2$, *and* $0 \leq \rho_{\pi,\pi^e} < 1$. *MI and CE can be used to study the dependence between Gaussian variables with different variance* σ^2 *and correlation* ρ_{π,π^e}. *However, they provide different information about the relationship between the variables. The plots show that MI is ordered across variance. Therefore, MI is decreasing in variance. It also shows that MI increases as correlation rises. The converse holds for CE. In other words, their MI decreases as the variables become more spread around the mean and less related. In contrast, the CE decreases as the correlation increases. This means that as the variables become highly correlated, the entropy of their copula decreases.*

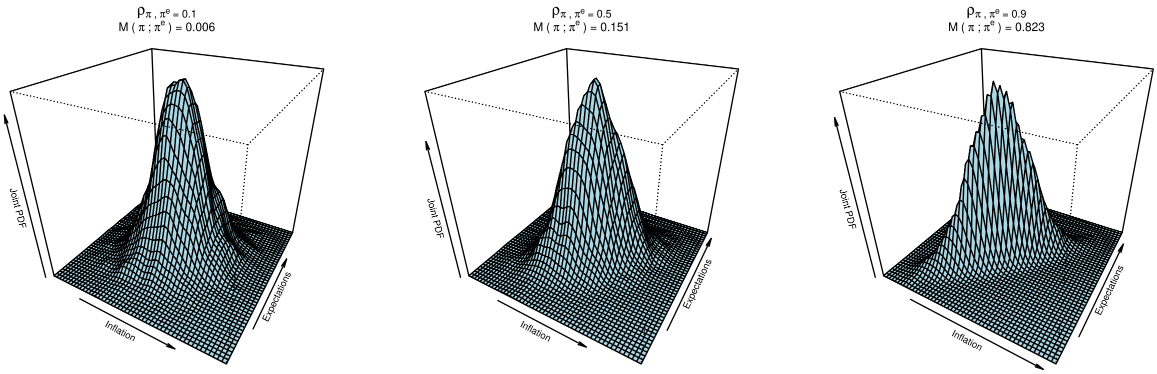

Figure 3: Joint PDFs and mutual information of two simulated Gaussians ($n = 10{,}000$).

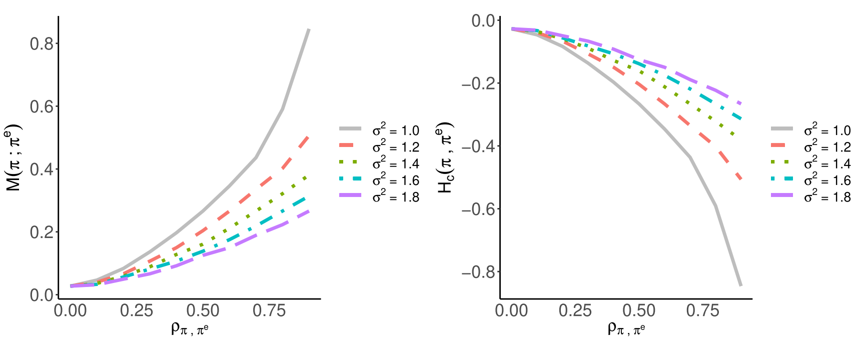

Figure 4: Mutual information $\mathcal{M}(\pi;\pi^e)$ and copula entropy $\mathcal{H}_c(\pi,\pi^e)$ for bivariate Gaussians with $\mu = 0, 1 \leq \sigma^2 < 2$, and $0 \leq \rho_{\pi,\pi^e} < 1$.

Therefore, Figure 4 *suggests that the mutual information and copula entropy provide different information about the relationship between the variables. They can both be used to study dependence but provide complementary perspectives.*

Lemma 1 has implications for understanding the sources of uncertainty in their relationship and can help to identify the effectiveness of monetary policy in achieving inflation targets beyond traditional methods (Ardakani, Kishor, and Song 2018). A higher copula entropy implies a higher level of uncertainty in the joint distribution, indicating a weaker degree of dependence between the two variables that are not explained by their marginal distributions. In other words, a higher level of uncertainty in the joint distribution of inflation and inflation expectations leads to less information shared between the two variables, indicating a weaker relationship. This weaker dependence may make it more challenging for central banks to anchor inflation expectations and achieve their targets. When inflation expectations are less anchored, individuals and firms may adjust their behavior in response to higher inflation uncertainty, leading to higher inflation rates and potentially even higher inflation expectations (Ardakani 2022b). Thus, it is essential for central banks to monitor the joint dynamics of inflation and inflation expectations closely and to take appropriate policy actions to maintain the stability of inflation expectations and ensure that they remain anchored to the central bank's inflation target. By doing so, the central bank can maintain its credibility and effectiveness in controlling inflation and promoting macroeconomic stability.

Proposition 1. *Let* $c\left(u_{\pi}, u_{\pi^e}\right)$ *represent the copula density function for the joint distribution of inflation* π *and inflation expectations* π^e. *The copula entropy* $\mathcal{H}_c(\pi,\pi^e)$, *given by Equation* (13), *quantifies the uncertainty in this* *joint distribution. A larger $\mathcal{H}_c(\pi, \pi^e)$ indicates a more uncertain and weaker dependence structure between π and π^e. This suggests that the predictive power of inflation expectations regarding actual inflation may diminish as the dependence weakens, presenting challenges for monetary policy in terms of price stabilization.*

Proof is provided in Appendix A.4. This proposition focuses on the dependence structure between inflation and expectations and its implications for monetary policy. Kullback (1997) shows the second derivative of the KLD with respect to the density functions parameters gives the Fisher information. Fisher information measures the amount of information a random variable contains about an unknown parameter (Fisher 1922).

Definition 2. *Fisher information: The Fisher information is defined by*

$$\begin{aligned}\mathcal{I}(\theta) &= \mathbb{E}\left(\frac{\partial}{\partial\theta}\log f(\pi, \pi^e; \theta)\right)^2 \\ &= -\mathbb{E}\left(\frac{\partial^2 \log f(\pi, \pi^e; \theta)}{\partial\theta^2}\right) \\ &= \text{Var}\left(\frac{\partial}{\partial\theta}\log f(\pi, \pi^e; \theta)\right),\end{aligned} \tag{16}$$

where θ is the parameter of the joint distribution $f(\pi, \pi^e)$.

The following theorem connects the Fisher information to the copula entropy.

Theorem 1. *For a multivariate distribution function, the copula entropy is bounded from below by the negative Fisher information, i.e.,*

$$\mathcal{H}_c(\pi, \pi^e) \geq -\mathcal{I}(\theta). \tag{17}$$

The proof is provided in Appendix A.5. Theorem 1 establishes a connection between copula entropy and Fisher information and has several implications:

1. It provides a lower bound for the copula entropy, implying a minimum uncertainty in the dependence structure between inflation and inflation expectations captured by copula entropy.
2. The Fisher information measures the amount of information contained in the parameters of the joint distribution. A larger Fisher information indicates that the parameters carry more information about the dependence between inflation and expectations. Therefore, a larger negative Fisher information implies a stronger dependence between the variables, which implies a smaller copula entropy.
3. The connection between copula entropy and Fisher information has policy implications for central banks. If the copula entropy is high central banks may need to take into account the weaker degree of dependence between the two variables. This may require additional measures or policies to effectively anchor inflation expectations and achieve inflation targets.

To illustrate Theorem 1, the following simulation study is conducted. First, a sample of bivariate normal data is generated ($n = 1000$) with known parameters μ_i, μ_j, covariance matrix Σ, and correlation ρ. Then, the copula entropy $\mathcal{H}_c(\pi, \pi^e)$ and Fisher information $\mathcal{I}(\theta)$ are estimated. The Fisher information matrix for a random variable $x \sim \mathcal{N}(\mu, \sigma^2)$ can be expressed as:

$$\mathcal{I}(\mu) = \frac{1}{\sigma^2} = \tau, \tag{18}$$

where τ is the precision or concentration parameter, defined as the reciprocal of the variance. Equation (18) is computed from the normal log-likelihood:

$$\log f(x; \mu) = -\log(\sqrt{2\pi\sigma^2}) - \frac{1}{2\sigma^2}(x-\mu)^2. \tag{19}$$

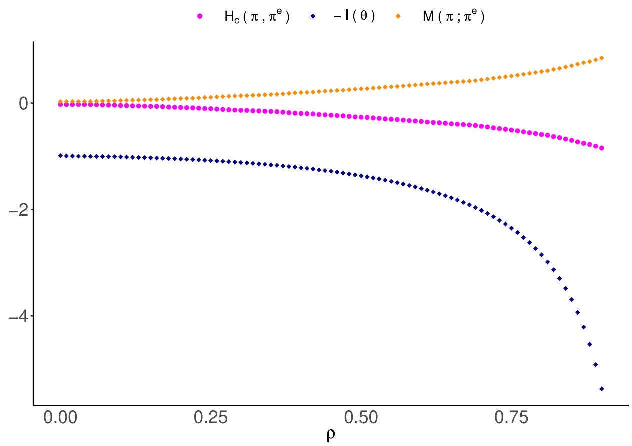

Figure 5: Lower and upper bounds of copula entropy for a bivariate normal ($n = 1000$). The red points represent copula entropy $\mathcal{H}_c(\pi, \pi^e)$, the blue points are the lower bound $-\mathcal{I}(\theta)$ shown in Theorem 1, and the orange points represent the mutual information $\mathcal{M}(\pi; \pi^e)$.

Hence, the partial derivative of the log-likelihood with respect to μ is:

$$\frac{\partial}{\partial \mu} \log\, f(x; \mu) = \frac{(x-\mu)}{\sigma^2}.$$

For a bivariate normal distribution $X \sim \mathcal{N}(\mu, \Sigma)$, where $\Sigma = \mathbb{E}[(X_i - \mu_i)(X_j - \mu_j)]$ is the covariance matrix, the Fisher information matrix can be expressed as:

$$\mathcal{I}(\mu) = -\mathbb{E}\left[\frac{\partial^2}{\partial \mu_i \mu_j}\left(-\frac{1}{2}\log|2\pi\Sigma| - \frac{1}{2}(X-\mu)'\Sigma^{-1}(X-\mu)\right)\right] = \left(\Sigma^{-1}\right)_{ij}. \tag{20}$$

The results are illustrated in Figure 5, which shows the copula entropy $\mathcal{H}_c(\pi, \pi^e)$ (represented by red points), the lower bound $-\mathcal{I}(\theta)$ (represented by blue points) as discussed in Theorem 1, and the mutual information $\mathcal{M}(\pi; \pi^e)$ (represented by orange points). This simulation example demonstrates the connection between copula entropy and Fisher information for a bivariate normal distribution. The approach presented in this section offers a framework that encapsulates mutual dynamics to capture dependencies and relationships in multivariate data. This framework is practical when studying tail events or extreme values as it examines joint behavior beyond marginal distributions.

4 Inflation and Its Expectations Dynamics

The interdependence between inflation and expectations is crucial in economics. It has significant implications for monetary policy and macroeconomic outcomes. In recent years, it has gained increasing attention from researchers and policymakers alike. As Bernanke (2007) noted, the state of inflation expectations influences inflation and the central bank's ability to achieve price stability. A growing body of literature has explored various aspects of this relationship, ranging from the estimation of expected inflation rates using survey data (Batchelor and Orr 1988; Carlson and Parkin 1975) to examining the rationality of inflation expectations (Andolfatto, Hendry, and Moran 2008), and analyzing the link between inflation and long-run inflation expectations (Chan, Clark, and Koop 2018). In addition, theoretical models incorporate inflation expectations as a key determinant of inflation, which can be influenced by central bank communications and policy actions (Friedman 1968; Lucas 1970; Phelps 1967). Central bank credibility and effective monetary policy are influenced by the relationship between inflation and expectations, as highlighted in Cruijsen and Demertzis (2007) and Rudd (2022).

The insights from recent studies highlight the correlation between actual inflation and short-run inflation expectations and invite deeper exploration into the causal underpinnings of this relationship. Understanding the relationship between actual inflation and short-run inflation expectations is pivotal for theoretical development and policy implications. D'Acunto et al. (2019) and D'Acunto and Weber (2022) point towards the significant role of immediate economic experiences and personal memory in shaping expectations. This suggests that short-run inflation expectations are not merely reactive but are formed based on the complexities of recent economic realities and individual psychological factors.

In addition, Coibion and Gorodnichenko (2015) examine the role of inflation expectations in the Phillips curve, further implying that short-run inflation expectations can significantly impact macroeconomic phenomena. Moreover, Garcia-Lembergman et al. (2023) emphasize the role of social networks and interactions in shaping inflation expectations. This perspective sheds light on the social and communal aspects of expectation formation, suggesting that short-run expectations are not only individual reactions but also a result of collective social processes. In this context, this paper contributes to understanding the short-run relationship between actual inflation and inflation expectations by employing a copula-based model. The proposed approach allows us to quantitatively analyze the dependence structure and the mutual information content between these two variables, offering how short-run inflation expectations are formed and evolve. This section builds upon these insights to provide empirical evidence of this dependence and the policy implications surrounding this relationship.

The empirical analysis encompasses the following data. Inflation is calculated using the Consumer Price Index (CPI) and the Producer Price Index (PPI). The CPI measures the prices of goods and services consumed by households, while the PPI measures the selling prices received by producers. Both indices are calculated by the U.S. Bureau of Labor Statistics. In addition to the traditional inflation metrics, the following sources gauge inflation expectations. The Surveys of Consumers at the University of Michigan gather data on the median expected price change over the next 12 months. This data provides a measure of how households expect prices to change in the near future.

Note that the median values from the University of Michigan's survey data are used instead of the mean. This choice is predicated on the robustness of the median to extreme values, ensuring that the findings are not influenced by outliers more prevalent in mean calculations. While the mean can offer insights, especially in stable economic conditions, the median provides a more reliable measure of central tendency in scenarios where extreme predictions might skew expectations. This consideration helps accurately capture the consensus view among consumers, particularly relevant in the context of short-term policy impact assessment. The Federal Reserve Bank of Cleveland also collects data on 2-, 10-, and 30-year expected inflation rates. These estimates are calculated using various sources, including Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations. Table 2 provides the summary statistics of inflation and expectations data. The monthly data range from January 1982 to November 2022, and percent changes from a year ago are considered for the empirical analysis. Figure 6 also plots the data.

Inflation data include deterministic and random components. Conventionally, inflation can be additively decomposed (Ardakani and Kishor 2018). This decomposition method involves computing the trend, cycle, and seasonal components. The remainder is calculated by subtracting the estimated seasonal and trend-cycle components. As noted by Hyndman and Athanasopoulos (2021), classical decomposition has several problems. First,

Table 2: Summary statistics.

	Mean	SD	25 %	50 %	75 %
π_{CPI}	2.90	1.66	1.76	2.75	3.73
π_{PPI}	2.59	5.55	−0.29	2.15	4.96
π^e_{UM}	3.14	0.69	2.70	3.00	3.30
π^e_{2YR}	2.80	1.21	1.81	2.65	3.52
π^e_{10YR}	2.84	1.09	1.92	2.59	3.57
π^e_{30YR}	2.87	0.73	2.22	2.72	3.35

The monthly data range from January 1982 to November 2022, and percent changes from a year ago are considered.

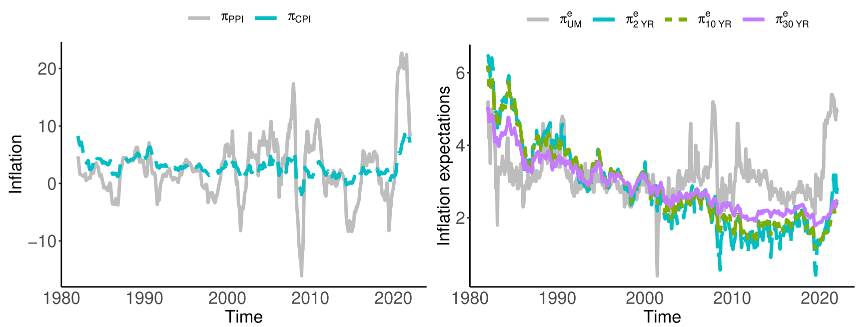

Figure 6: Inflation and inflation expectations. Inflation: π_{CPI} (Consumer Price Index) and π_{PPI} (Producer Price Index); Inflation expectations: π^e_{UM} (University of Michigan's Surveys of Consumers), π^e_{2YR}, π^e_{10YR}, and π^e_{30YR} (2-, 10-, and 30-year expectations).

the first few and last few observations are lost. Second, the trend-cycle estimate tends to oversooth the fluctuation in the data. Third, they do not capture the seasonal changes over time since the seasonal component repeats yearly. An alternative method is the X-11, which is considered the proper decomposition technique (Dagum and Bianconcini 2016). The updated version of this method is known as X-13ARIMA-SEATS or X-13 (Monsell 2007). The details on implementation can be found in Sax and Eddelbuettel (2018). This method is robust to outliers so that the outliers will not affect the estimates of trend, cycle, and seasonal components. Figure 7 plots the random components of inflation and inflation expectations.

Table 3 presents copula density estimates for the relationship between inflation and inflation expectations. The table lists copulas with different combinations of inflation measures (PPI and CPI) and inflation expectations (UM, 2YR, 10YR, and 30YR). The copulas are denoted by $c(u_\pi, u_{\pi^e})$, where u_π and u_{π^e} are the transformed inflation and inflation expectations measures, respectively, to have uniform marginal distributions. The table provides the following information for each copula. Bandwidth gives the bandwidth used to estimate the copula density. Log-likelihood, AIC, and BIC provide the log-likelihood, Akaike information criterion, and Bayesian information criterion values of the copula density estimate. Kendall's τ provides the correlation coefficient, a measure of the strength and direction of association between inflation and inflation expectation. Kendall's τ is functional of the copula and can be expressed as

$$\tau = 4 \int_{[0,1]^2} C(u_\pi, u_{\pi^e}) c(u_\pi, u_{\pi^e}) \mathrm{d}u_\pi \mathrm{d}u_{\pi^e} - 1. \tag{21}$$

The integral is numerically solved to calculate τ in Equation (21). The findings illustrate the comovement of inflation and expected inflation.

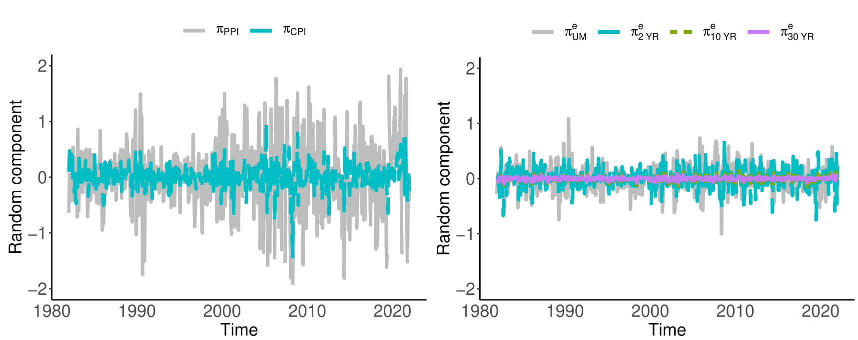

Figure 7: Random components of inflation and expectations. Inflation: π_{CPI} (Consumer Price Index) and π_{PPI} (Producer Price Index); Inflation expectations: π^e_{UM} (University of Michigan's Surveys of Consumers), π^e_{2YR}, π^e_{10YR}, and π^e_{30YR} (2-, 10-, and 30-year expectations).

Table 3: Copula density estimates for inflation and inflation expectations.

	Bandwidth	Log-likelihood	AIC	BIC	Kendall's τ
$c\left(u_{\pi_{PPI}}, u_{\pi^e_{UM}}\right)$	0.57	24.42	−24.66	26.08	0.16
$c\left(u_{\pi_{PPI}}, u_{\pi^e_{2YR}}\right)$	0.50	19.08	−11.59	44.18	0.11
$c\left(u_{\pi_{PPI}}, u_{\pi^e_{10YR}}\right)$	0.57	16.48	−8.83	41.79	0.13
$c\left(u_{\pi_{PPI}}, u_{\pi^e_{30YR}}\right)$	0.57	12.13	−0.42	49.60	0.11
$c\left(u_{\pi_{CPI}}, u_{\pi^e_{UM}}\right)$	0.54	17.74	−9.96	43.60	0.12
$c\left(u_{\pi_{CPI}}, u_{\pi^e_{2YR}}\right)$	0.57	19.79	−15.01	36.54	0.14
$c\left(u_{\pi_{CPI}}, u_{\pi^e_{10YR}}\right)$	0.57	13.93	−3.31	48.18	0.10
$c\left(u_{\pi_{CPI}}, u_{\pi^e_{30YR}}\right)$	0.57	10.31	3.81	55.06	0.08

The copula density estimates are reported for the relationship between inflation measures (CPI and PPI) and expectations (UM for University of Michigan Surveys, 2YR, 10YR, and 30YR for 2-, 10-, and 30-year). The copulas $c(u_\pi, u_{\pi^e})$ are estimated using bandwidth optimization with u_π and u_{π^e} representing the uniform marginals. AIC is the Akaike Information Criterion, and BIC is the Bayesian Information Criterion. Kendall's τ is the non-parametric correlation coefficient derived from the copula.

Figure 8 also displays the densities of copulas for inflation and inflation expectations. The top row shows copula densities calculated using the PPI to measure inflation, while the bottom row presents copula densities based on the CPI. Kernel density estimation is utilized to estimate the copula density. It calculates the joint density of two variables based on their marginal densities and copula density. The marginal densities are used to estimate copula densities. Specifically, the copula density is approximated by a smoothing function (the kernel) placed at each data point. The smoothing parameter, or bandwidth, determines the amount of smoothing applied to the data and affects the shape of the copula density estimate. This procedure is considered an accurate non-parametric copula density estimation Nagler (2018). Assume we have *iid* observations $\left(U_{i,\pi}, U_{i,\pi^e}\right),\ i = 1, \ldots, n$ from a bivariate copula C. The kernel density estimator for copula density $c\left(u_\pi, u_{\pi^e}\right)$ is given by

$$\hat{c}\left(u_\pi, u_{\pi^e}\right) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{u_\pi - U_{i,\pi}}{h}\right)K\left(\frac{u_{\pi^e} - U_{i,\pi^e}}{h}\right), \quad \left(u_\pi, u_{\pi^e}\right) \in [0,1]^2, \tag{22}$$

where K is the non-negative kernel function, and h is the bandwidth parameter. This estimator suffers from bias at the boundaries (Charpentier, Fermanian, and Scaillet 2006). This bias refers to the issue that the estimated copula function $\hat{c}$ does not always accurately represent the underlying copula function c. This is because the estimated $\hat{c}$ does not integrate to one, meaning it does not satisfy the requirements of being a density function. Additionally, $\hat{c}$ is not a density function on the unit square $[0,1]^2$. The transformation method of local likelihood estimation is used to mitigate this bias. This method transforms the data to a support on the two-dimensional real space $\mathbb{R}^2$ rather than the unit cube, reducing the bias at the boundaries. This transformation method is described in detail in (Geenens, Charpentier, and Paindaveine 2017).

The findings suggest that copula densities exhibit heavy-tailed behavior. A heavy-tailed copula indicates higher uncertainty in the relationship between inflation and expectations. In essence, the joint distribution has a more pronounced dependence on the distribution's tails, where extreme values are found. This phenomenon of tail dependence is a focal topic in extreme value theory and is especially significant in understanding extreme events and tail risks (Ardakani 2023a). Archimedean copulas better capture the tail dependence observed in the data (Ledford 1996). The finding underscores an "extreme" positive relationship between inflation and inflation expectations at the tails of their joint distribution. This indicates a heightened dependence level for extreme high and low values, akin to the ideas presented in coherent portfolio risk measures (Ardakani 2023b).

Information measures defined in Equations (6)–(11) are presented in Table 4. This finding gives insights into the information content of expected inflation. The negative entropy of expected inflation across all measures indicates that it contains information that can be leveraged for inflation forecasting. This is supported by

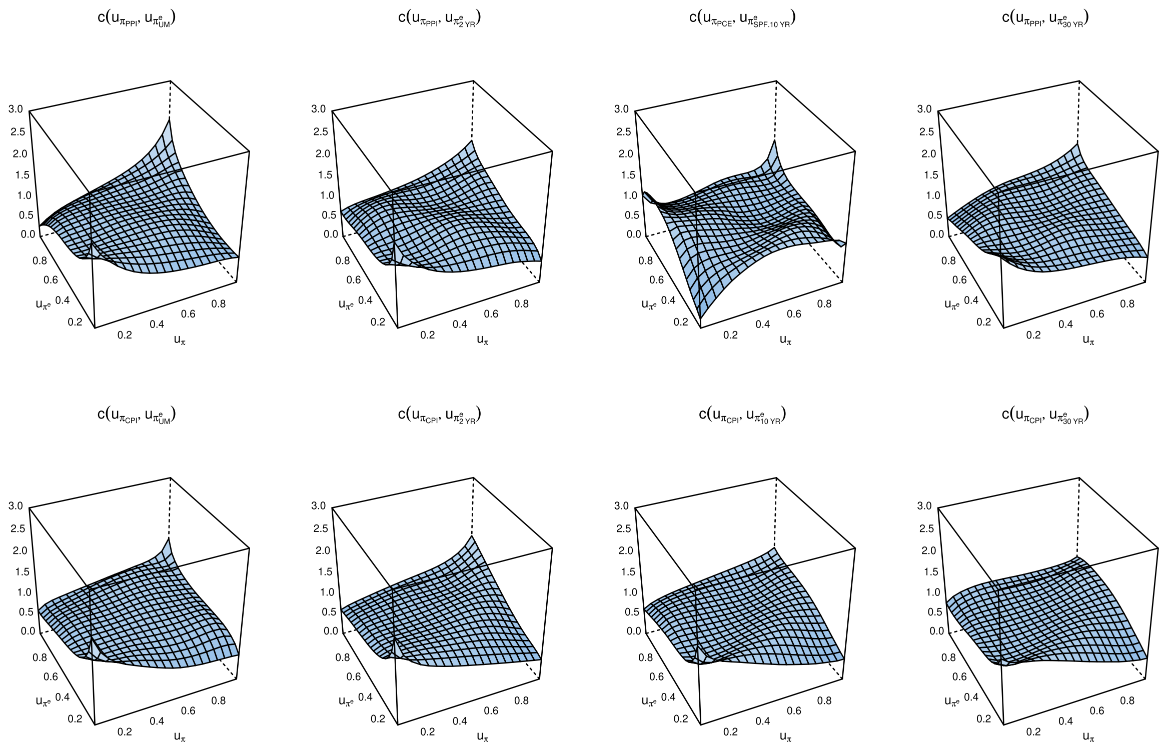

Figure 8: Perspective plots of copula density for inflation and inflation expectations. The copula density estimates are plotted for the relationship between inflation measures (CPI and PPI) and expectations (UM for University of Michigan Surveys, 2YR, 10YR, and 30YR for 2-, 10-, and 30-year). The copulas $c(u_\pi, u_{\pi^e})$ are estimated using bandwidth optimization with u_π and u_{π^e} representing the uniform marginals.

Table 4: Information measures for inflation and inflation expectations.

	$\mathcal{H}(\pi^e)$	PPI			CPI		
		$\mathcal{H}(\pi,\pi^e)$	$\mathcal{K}(\pi;\pi^e)$	$\mathcal{M}(\pi;\pi^e)$	$\mathcal{H}(\pi,\pi^e)$	$\mathcal{K}(\pi;\pi^e)$	$\mathcal{M}(\pi;\pi^e)$
π^e_{UM}	−0.156	1.904	0.757	0.014	−0.027	0.158	0.014
$\pi^e_{2\text{YR}}$	−0.271	2.122	0.975	0.031	−0.072	0.113	0.012
$\pi^e_{10\text{YR}}$	−1.675	4.776	3.629	0.031	1.643	1.829	0.024
$\pi^e_{30\text{YR}}$	−2.280	5.320	4.173	0.002	2.687	2.872	0.013

Expectation entropy $\mathcal{H}(\pi^e)$, joint entropy $\mathcal{H}(\pi,\pi^e)$, KL divergence $\mathcal{K}(\pi;\pi^e)$, and mutual information $\mathcal{M}(\pi;\pi^e)$ of inflation and expectations are reported.

the fact that the entropy of the uniform distribution is zero, implying that expected inflation carries meaningful information beyond randomness. Moreover, the Kullback-Leibler divergence, which measures the discrepancy between the inflation density and expected inflation, increases from π^e_{UM} to $\pi^e_{30\text{YR}}$, suggesting a greater divergence from the inflation density. This consistent trend is observed for both PPI and CPI. Additionally, the mutual information, which quantifies the dependence between inflation and expected inflation, is found to be lowest for $\pi^e_{30\text{YR}}$. These findings highlight the importance of expected inflation as an informative variable for inflation forecasting, mainly when using longer-term expectations such as $\pi^e_{30\text{YR}}$.

Figure 9 compares joint entropy, Kullback-Leibler divergence, and mutual information as information-theoretic measures used to analyze the relationship between inflation and inflation expectations over an

Figure 9: Joint entropy $\mathcal{H}(\pi, \pi^e)$, Kullback-Leibler divergence $\mathcal{K}(\pi; \pi^e)$, and mutual information $\mathcal{M}(\pi; \pi^e)$ of inflation and inflation expectations on expanding windows.

expanding window. These measures provide insights into the dynamic relationship, shedding light on how they evolve. Cross-entropy measures the difference between the distribution of inflation and the distribution of inflation expectations, offering a quantitative assessment of the disparity between the two. The Kullback-Leibler divergence captures the asymmetry between the two distributions, quantifying how much one distribution diverges from the other. On the other hand, mutual information measures the total correlation between the two variables, offering a holistic view of their relationship. By examining these three measures over time, Figure 9 visualizes the changing relationship between inflation and inflation expectations.

Table 5 provides information on the average information-theoretic measures of inflation and inflation expectations based on the two sources of inflation data (PPI and CPI) and four sources of inflation expectations (the University of Michigan survey, the Federal Reserve Bank of Cleveland 2-year, 10-year, and 30-year expectations). The three information-theoretic measures used in the table are average cross-entropy ($\bar{\mathcal{H}}$), Kullback-Leibler ($\bar{\mathcal{K}}$), and mutual information ($\bar{\mathcal{M}}$). The results show the average values of these measures for each combination, providing insights into the quantitative assessments of the disparity, asymmetry, and total correlation between inflation and inflation expectations. The following points are noteworthy:

1. The cross-entropy values indicate that the University of Michigan survey and the Federal Reserve Bank of Cleveland's 2-year expectations better predict current inflation than the 10-year and 30-year expectations. This may suggest that these shorter-term expectations are more closely tied to the current inflation rate.
2. On average, the 10-year and 30-year Federal Reserve Bank of Cleveland expectations have the highest Kullback-Leibler values, indicating that these sources provide the least similar information about the current and expected inflation rates. This finding indicates that these longer-term expectations are more influenced by factors beyond the current inflation rate, such as economic growth expectations or monetary policy.
3. The CPI and PPI inflation data sources produce similar results, with the CPI showing slightly lower cross-entropy and Kullback-Leibler values than the PPI. This suggests that the choice of inflation data source may not have a significant impact on the conclusions drawn from the information-theoretic measures.

Table 5: Average information-theoretic measures.

		$\bar{\mathcal{H}}(\pi, \pi^e)$	$\bar{\mathcal{K}}(\pi;\pi^e)$	$\bar{\mathcal{M}}(\pi;\pi^e)$
PPI (π_{PPI})	π^e_{UM}	0.038	0.706	0.038
	π^e_{2YR}	0.024	0.949	0.024
	π^e_{10YR}	0.020	2.978	0.020
	π^e_{30YR}	0.013	3.434	0.013
CPI (π_{CPI})	π^e_{UM}	0.015	0.113	0.015
	π^e_{2YR}	0.021	0.044	0.021
	π^e_{10YR}	0.004	1.454	0.004
	π^e_{30YR}	0.003	2.294	0.003

Average joint entropy $\bar{\mathcal{H}}(\pi, \pi^e)$, KL divergence $\bar{\mathcal{K}}(\pi; \pi^e)$, and mutual information $\bar{\mathcal{M}}(\pi; \pi^e)$ of inflation and expectations are reported.

Robustness checks are also conducted using alternative inflation expectations and actual inflation measures to validate these findings. Specifically, data from the Survey of Professional Forecasters for both short-term (1-year) and long-term (10-year) inflation expectations, alongside the Personal Consumption Expenditures (PCE) inflation data, are included. This allows us to compare and contrast the main results with those obtained using different measures. The detailed results of this supplementary analysis are presented in Equation (A.6).

Integrating copula-based models with information theory provides valuable insights into the dynamics between inflation and expectations, which has significant implications for monetary policy and macroeconomic outcomes. This approach takes into account the joint distribution of inflation and expectations and captures their dependence. The evidence underscores the need for central banks to recognize the role of expectations in shaping inflation dynamics. Anchoring inflation expectations at the desired level is crucial for maintaining central bank credibility and ensuring effective monetary policy.

5 Concluding Remarks

This paper proposes a framework for examining the information content of inflation expectations by integrating copula modeling and information-theoretic measures. Copulas are employed to estimate the dynamic dependence between inflation and expectations, enabling analysis of extreme events and tail dependence. Information-theoretic measures are then used to quantify the information expected inflation holds about the inflation rate. The contribution of this paper is the link established between copula entropy and information-theoretic measures, which provides a powerful tool for central banks to anchor inflation expectations to achieve inflation targets effectively. The theoretical result establishes a lower bound for the copula entropy, implying a minimum degree of uncertainty in the dependence between inflation and expectations. This finding has significant policy implications, providing insights into the uncertainty expected in the relationship between inflation and expectations.

The empirical findings suggest that the copula densities are heavy-tailed, implying higher uncertainty in the tails of the distribution. The results also show that expected inflation carries meaningful information for forecasting inflation, mainly when using longer-term expectations. The information-theoretic measures provide quantitative assessments of the disparity, asymmetry, and total correlation between inflation and expectations. Furthermore, the results indicate that the University of Michigan survey and the Federal Reserve Bank of Cleveland's 2-year expectations provide better predictions of current inflation than longer-term expectations. These findings have implications for risk management, policy decisions, and further research. Further research can build on these findings to deepen our understanding of the role of expected inflation in inflation forecasting and its implications for monetary policy strategies.

The proposed framework offers several advantages. First, it offers a tool to analyze extreme events and tail dependence and captures asymmetric behaviors typically hidden in averaged or aggregated metrics. The second benefit lies in the ability of copula-based models to scale to higher dimensions. While this paper focuses on the bivariate relationship, the methodology is not restricted to this. It can be extended to study multiple outcomes simultaneously to examine economic interactions. Third, the model's emphasis on copula densities facilitates robust density forecasting. We can make informed predictions by focusing on the entire distribution rather than point forecasts. This is pivotal when the likelihood of extreme outcomes is as crucial as predicting the most probable outcome.

By highlighting tail dependencies, the proposed framework helps policymakers identify and preempt extreme economic events. The quantitative assessment of disparity, asymmetry, and correlation between inflation and expectations provides a foundation for a higher degree of transparency by central banks. Policymakers can address specific misconceptions or expectations in the market. The differential analysis between short-term and long-term expectations and their forecasting power can guide central banks' policy decisions. By understanding which expectations provide more predictive power, policymakers can fine-tune their interventions to be more impactful.

Several avenues for future research are recognized. Expanding this analysis to include longer-term inflation expectations would offer a broader perspective, especially in understanding how these expectations evolve in response to changing economic policies and conditions. Investigating the interaction between short-term and long-term forecasts could clarify the dynamics of expectation formation over different time horizons. Secondly, comparing median and mean inflation expectations can provide insights into statistically measuring consumer sentiment. This exploration could reveal how different statistical measures respond under varied economic scenarios, potentially leading to more robust forecasting models. Moreover, future studies could examine sector-specific inflation expectations to understand how different economic sectors perceive and react to inflationary trends. This sectoral analysis might uncover unique patterns and dependencies that are not apparent in the aggregate data. Lastly, a cross-country comparative study could provide valuable insights into how different economic systems and central bank policies influence inflation expectations. Such an analysis would contribute to a global understanding of inflation dynamics.

Appendix: Proofs

A.1 Mutual Information Relation to Entropy

$$\begin{aligned}
\mathcal{M}(\pi;\pi^e) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\big(f(\pi^e)\big)\mathrm{d}\pi\,\mathrm{d}\pi^e \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi)f(\pi^e|\pi)\log\big(f(\pi^e|\pi)\big)\mathrm{d}\pi\,\mathrm{d}\pi^e - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\big(f(\pi^e)\big)\mathrm{d}\pi\,\mathrm{d}\pi^e \\
&= \int_{-\infty}^{\infty} f(\pi)\mathrm{d}\pi\left(\int_{-\infty}^{\infty} f(\pi^e|\pi)\log\big(f(\pi^e|\pi)\big)\mathrm{d}\pi^e\right) - \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\pi,\pi^e)\mathrm{d}\pi\right)\log(f(\pi^e))\mathrm{d}\pi^e \\
&= -\int_{-\infty}^{\infty} f(\pi)\mathrm{d}\pi\mathcal{H}(\pi^e|\pi) - \int_{-\infty}^{\infty} f(\pi^e)\log\big(f(\pi^e)\big)\mathrm{d}\pi^e \\
&= -\mathcal{H}(\pi^e|\pi) + \mathcal{H}(\pi^e) = \mathcal{H}(\pi^e) - \mathcal{H}(\pi^e|\pi).
\end{aligned}$$

Similarly, we can show that $\mathcal{M}(\pi;\pi^e) = \mathcal{H}(\pi) - \mathcal{H}(\pi|\pi^e)$.

A.2 Mutual Information Relation to KLD

$$\mathcal{M}(\pi;\pi^e) = \mathcal{K}\big(f(\pi,\pi^e){:}\, f(\pi)f(\pi^e)\big) = \mathbb{E}_{\pi^e}\big[\mathcal{K}\big(f(\pi|\pi^e){:}\, f(\pi)\big)\big]$$

$$\begin{aligned}
\mathcal{M}(\pi;\pi^e) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi|\pi^e)f(\pi^e)\log\left(\frac{f(\pi|\pi^e)f(\pi^e)}{f(\pi)f(\pi^e)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e
\end{aligned}$$

$$= \int_{-\infty}^{\infty} f(\pi^e)\mathrm{d}\pi^e \int_{-\infty}^{\infty} f(\pi|\pi^e)\log\left(\frac{f(\pi|\pi^e)}{f(\pi)}\right)\mathrm{d}\pi$$

$$= \int_{-\infty}^{\infty} f(\pi^e)\mathrm{d}\pi^e \mathcal{K}\big(f(\pi|\pi^e): f(\pi)\big)$$

$$= \mathbb{E}_{\pi^e}\big[\mathcal{K}\big(f(\pi|\pi^e): f(\pi)\big)\big].$$

A.3 Proof of Lemma 1

$$\mathcal{M}(\pi;\pi^e) = -\mathcal{H}_c(\pi,\pi^e).$$

A two-dimensional copula is a bivariate CDF defined on the uit cube with uniform marginal distributions on the interval [0,1] and can be defined as

$$C\big(u_\pi, u_{\pi^e}\big) = F(\pi,\pi^e) = F\big(F^{-1}(u_\pi), F^{-1}\big(u_{\pi^e}\big)\big).$$

The copula density function can be derived as

$$c\big(u_\pi;, u_{\pi^e}\big) = \frac{\partial^2 C\big(u_\pi, u_{\pi^e}\big)}{\partial u_\pi \partial u_{\pi^e}} = \frac{\partial^2 C(F(\pi), F(\pi^e))}{\partial F(\pi)\partial F(\pi^e)}$$
$$= \frac{\partial^2 F(\pi,\pi^e)}{f(\pi)f(\pi^e)\partial\pi\partial\pi^e} = \frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}$$

$$\mathcal{M}(\pi;\pi^e) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\pi,\pi^e)\log\left(\frac{f(\pi,\pi^e)}{f(\pi)f(\pi^e)}\right)\mathrm{d}\pi\,\mathrm{d}\pi^e$$
$$= \iint_{[0,1]^2} c\big(u_\pi, u_\pi^e\big)\log\, c\big(u_\pi, u_\pi^e\big)\mathrm{d}u_\pi u_{\pi^e}$$
$$= -\mathcal{H}_c(\pi,\pi^e).$$

A.4 Proof of Proposition 1

The copula entropy $\mathcal{H}_c(\pi,\pi^e)$ quantifies the uncertainty in the joint distribution of inflation π and inflation expectations π^e, defined by the copula density function $c\big(u_\pi, u_{\pi^e}\big)$. Formally,

$$\mathcal{H}_c(\pi,\pi^e) = -\int_0^1\int_0^1 c\big(u_\pi, u_{\pi^e}\big)\log\, c\big(u_\pi, u_{\pi^e}\big)\,\mathrm{d}u_\pi\,\mathrm{d}u_{\pi^e},$$

where u_π and u_{π^e} are the uniform marginals obtained from the CDFs of π and π^e, respectively. A higher value of $\mathcal{H}_c(\pi,\pi^e)$ indicates a greater level of uncertainty in the joint distribution, reflecting a weaker dependence structure between π and π^e. Consider the conditional entropy of π given π^e, which measures the residual uncertainty in π after observing π^e. A weaker dependence, as indicated by a higher copula entropy, would result in a higher conditional entropy, signifying that knowing π^e reduces less uncertainty about π.

A.5 Proof of Theorem 1

For a multivariate distribution function, the Copula entropy is bounded from below by the negative Fisher information, i.e.,

$$\mathcal{H}_c(\pi, \pi^e) \geq -\mathcal{I}(\theta).$$

Using the Cramér-Rao bound, Equations (6) and (12), Brunel and Nadal (1998) show that the mutual information between π and π^e is upper bounded by the Fisher information of their joint distribution. That is,

$$\mathcal{M}(\pi; \pi^e) \leq \mathcal{I}(\theta).$$

The proof is provided below.

From Equation (6),

$$\mathcal{M}(\pi; \pi^e) = \mathbb{E}[\log\ f(\pi, \pi^e)] - \mathbb{E}[\log\ f(\pi)] - \mathbb{E}[\log\ f(\pi^e)].$$

By the definition of Fisher information,

$$\begin{aligned} I(\theta) &= \mathrm{Var}\left(\frac{\partial}{\partial\theta}\log\ f(\theta)\right) \\ &= \mathbb{E}\left(\frac{\partial}{\partial\theta}\log\ f(\theta)\right)^2. \end{aligned}$$

Using the chain rule for logarithms,

$$\log\ f(\pi, \pi^e) = \log\ f(\pi) + \log\ f(\pi^e) + \log\ f(\theta),$$

which implies that

$$\frac{\partial}{\partial\theta}\log\ f(\pi, \pi^e) = \frac{\partial}{\partial\theta}\log\ f(\theta) = \frac{1}{f(\theta)}\frac{\partial}{\partial\theta}f(\theta).$$

Substituting this expression into the definition of Fisher information, we obtain

$$I(\theta) = \mathrm{Var}\left(\frac{1}{f(\theta)}\frac{\partial}{\partial\theta}f(\theta)\right).$$

Using the Cauchy-Schwarz inequality,

$$\begin{aligned} \mathrm{Var}\left(\frac{1}{f(\theta)}\frac{\partial}{\partial\theta}f(\theta)\right)\cdot \mathrm{Var}(\log\ f(\pi, \pi^e)) &\geq \left(\mathbb{E}\left[\frac{1}{f(\theta)}\frac{\partial}{\partial\theta}f(\theta)\log\ f(\pi, \pi^e)\right]\right)^2 \\ &= \left(\mathbb{E}\left[\frac{\partial}{\partial\theta}\log\ f(\pi, \pi^e)\right]\right)^2 \\ &= \mathcal{M}^2(\pi; \pi^e). \end{aligned}$$

Therefore, we have

$$\mathcal{I}(\theta)\cdot\mathcal{M}(\pi; \pi^e) \geq \mathcal{M}^2(\pi; \pi^e),$$

which simplifies to

$$\mathcal{M}(\pi; \pi^e) \leq \mathcal{I}(\theta).$$

Also, from Lemma 1,

$$\mathcal{M}(\pi; \pi^e) = -\mathcal{H}_c(\pi, \pi^e).$$

Therefore, we have

$$\mathcal{H}_c(\pi, \pi^e) \geq -\mathcal{I}(\theta).$$

A.6 Robustness Checks

Various measures of inflation expectations and actual inflation are employed to gauge price movements and expectation trends. The Survey of Professional Forecasters (SPF) and Personal Consumption Expenditures (PCE) are prominent among these. The SPF provides inflation forecasts from professional economists, offering insights into future inflation trends. For this robustness check, I utilize the 1-year and 10-year forecasts from the SPF, which are provided on a quarterly basis. While the main results are based on monthly data, the analysis is adapted to the quarterly frequency of the SPF data in this section. These forecasts represent the professional consensus on short-term and long-term inflation expectations, respectively. On the other hand, PCE inflation data offers a measure of actual inflation based on consumer spending and is widely used in policy analysis. We can compare forecasted inflation with actual inflation outcomes by incorporating PCE data. This Appendix presents an empirical analysis using these alternative measures, adjusted to the quarterly frequency provided by the SPF.

The results are presented in Table A.1 and Figure A.1. The copula density estimates using the PCE and SPF data exhibit similar patterns to those observed with CPI, PPI, and the University of Michigan's survey data.

Table A.1: Copula density estimates for inflation and inflation expectations.

	Bandwidth	Log-likelihood	AIC	BIC	Kendall's τ
$c\left(u_{\pi_{\text{PCE}}}, u_{\pi^e_{\text{SPF.1YR}}}\right)$	0.63	6.15	8.46	40.89	−0.025
$c\left(u_{\pi_{\text{PCE}}}, u_{\pi^e_{\text{SPF.10YR}}}\right)$	0.54	10.89	1.21	37.12	−0.030

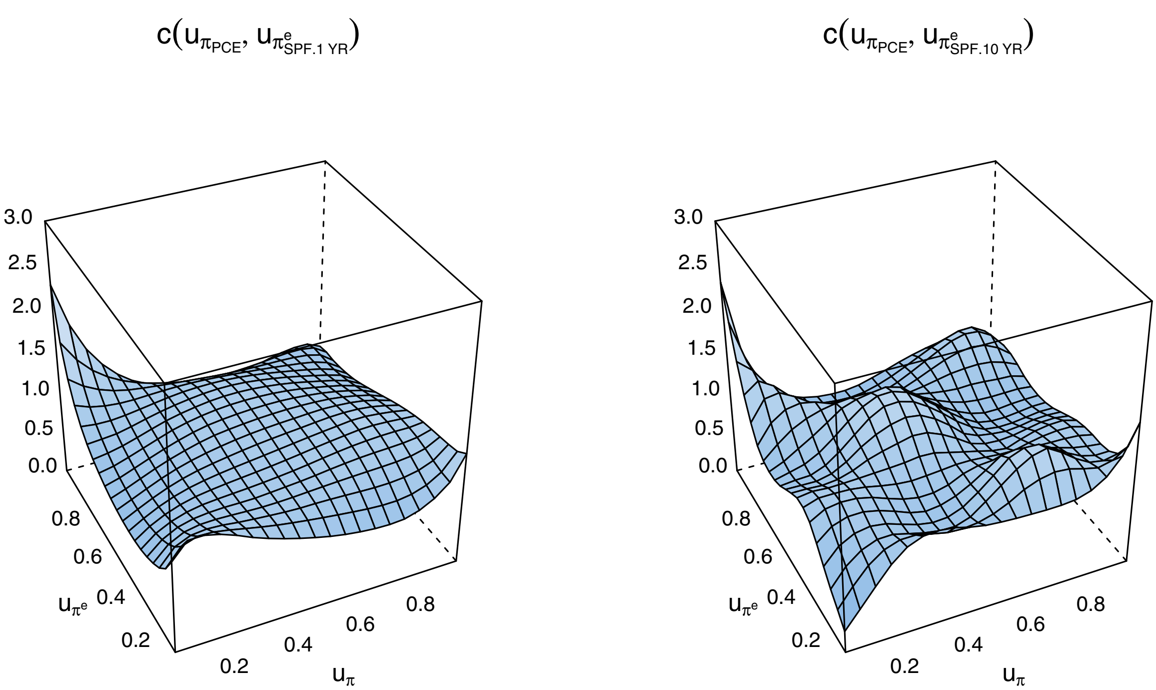

Figure A.1: Perspective plots of copula density for inflation and inflation expectations. The copula density estimates are plotted for the relationship between the inflation measure based on the Personal Consumption Expenditures and expectations based on the Survey of Professional Forecasters (1-year and 10-year). The copulas $c(u_\pi, u_{\pi^e})$ are estimated using bandwidth optimization with u_π and u_{π^e} representing the uniform marginals.

While Kendall's τ values for the SPF-based measures are negative, indicating a weak inverse relationship, this does not qualitatively change our main conclusions regarding the dynamics between inflation and expectations. The slight differences in the strength and direction of these associations underscore the multifaceted nature of inflation expectations and their complex relation to actual inflation. Overall, the consistency across different data sources reinforces the robustness of the main findings and supports the reliability of the copula-based approach in capturing the essence of this relationship.

The copula density estimates are reported for the relationship between inflation measures (CPI and PPI) and expectations (UM for University of Michigan Surveys, 2YR, 10YR, and 30YR for 2-, 10-, and 30-year). The copulas $c(u_\pi, u_{\pi^e})$ are estimated using bandwidth optimization with u_π and u_{π^e} representing the uniform marginals. AIC is the Akaike Information Criterion, and BIC is the Bayesian Information Criterion. Kendall's τ is the non-parametric correlation coefficient derived from the copula.

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