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On the comparison of inequality measures: evidence from the world values survey

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ABSTRACT
This paper overviews the well-known inequality measures and applies information-theoretic tools to compare inequality between income levels among nations. Specifically, we use the World Values Survey data on households' income levels in 57 countries to rank inequality. We then examine the effects of macroeconomic outcomes on inequality. Our findings suggest that the Gini coefficient with *J*-divergence provides further insight into inequality rankings. In addition, the results indicate that higher inflation and GDP growth lead to lower inequality. Finally, we find a positive relationship between income inequality and money growth.

KEYWORDS
Gini index; income inequality; Information-Theoretic measures

JEL CLASSIFICATION
D31; D63

I. Introduction

In recent years, most studies have shown rising inequality within developed and developing countries (Alvaredo et al. 2017; Kopczuk 2015). The literature on inequality has proposed estimates of income concentration (Piketty 2013; Saez and Zucman 2016; Zucman 2013) and discussed inequality through probabilistic frameworks (Cowell 2011; Dagum 1990; Maasoumi 1986). For example, the Gini coefficient derived from the Lorenz curve can be represented with the probability distribution function. Inequality can also be studied through information-theoretic measures, such as Shannon entropy, hereafter entropy. Entropy captures the information content of income. One strand of the literature has shown that increasing entropy is associated with increased wealth and developed inequality indices based on the notion of entropy (Koutsoyiannis and Sargentis 2021; Ryu 2013).

This paper provides examples to illustrate how entropy is used to capture inequality and is linked with Gini. We employ divergence measures between two probability distributions to compare the degree of inequality. The Kullback–Leibler divergence (Kullback 1959) measures the divergence between two income distributions.

However, this measure is asymmetric, resulting in different values of inequality for the same income distribution. We use the symmetric measure of *J*-divergence (Jeffreys 1946), measured by the average of two Kullback–Leibler distances. As information-theoretic tools, Kullback-Leibler and *J*-divergence can represent an inequality analogous to the role of the Lorenz curve in characterizing the inequality: moving away from a uniform income distribution means higher inequality. This representation captures inequality between income levels. We then utilize the World Values Survey data to construct the inequality measures and compare income inequality by including all the countries studied in Wave 7 (2017–2020). Specifically, we provide corresponding rankings using the proportions of low-, middle-, and high-income populations. Our findings suggest that the Gini coefficient with *J*-divergence provides a deeper understanding of inequality rankings. The Gini coefficient emphasizes the proportion of individuals in the low-income levels while *J*-divergence represents the statistical distance from equally distributed income levels.

We also examine the relationships between inequality and inflation, broad money growth, and GDP per capita growth within a Bayesian framework. Our findings suggest a positive relationship between money growth and inequality. The literature elaborates on this positive relationship. The expansionary monetary policy provides excess

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money to financial institutions which distribute the fund to wealthy investments, leading to higher inequality (Bagchi, Curran, and Fagerstrom 2019; Doepke and Schneider 2006; Meh, Ríos-Rull, and Terajima 2010). In addition, the results indicate a negative relationship between inflation and inequality. The negative relationship between inflation and inequality is expected when capital heterogeneity dominates skill heterogeneity (Jin 2009). We also find a negative relationship between GDP growth and inequality.

II. Inequality measures

The question of whether income is equally distributed within levels among nations has led to theoretical and empirical studies, including seminal works of Atkinson (1970), Champernowne (1974), Muellbauer (1974), Sheshinski (1972), and Yitzhaki (1983). The Gini coefficient (Gini 1912) is considered the primary measure of inequality and is estimated through various methods (Dorfman 1979; Milanovic 1997; Paglin 1975), such as using the cumulative distribution function (CDF). The CDF of income can be written as

$$F(y) = \int_0^{\infty} f(y)dy, \tag{1}$$

where $f(y)dF(y)/dy$ is the continuous probability density function (PDF). The mean of income distribution is given by $\mu = \int_0^{\infty} yf(y)dy$. The Gini coefficient, which is given by

$$\begin{aligned}\mathcal{G} &= 1 - \frac{1}{\mu}\int_0^{\infty}(1-F(y))^2 dx \\ &= \frac{1}{\mu}\int_0^{\infty} F(y)(1-F(y))dy,\end{aligned} \tag{2}$$

was initially derived from the Lorenz curve $\mathcal{L} = \int_0^{x} yf(y)dy/\mu$, that is a proportion of total income cumulatively earned by the bottom x of the population. The area between the equality line and the Lorenz curve provides the Gini coefficient, ranging from zero (perfect equality) to one (perfect inequality). Alternatively, inequality can be estimated by information-theoretic measures. Shannon entropy (Shannon 1948), defined as a measure of diversity, is associated with equality by measuring concentration in each income level (Mishra and Ayyub 2019). Entropy can be written as

$$\begin{aligned}\mathcal{H}(f) &= \mathbb{E}[\log(\frac{1}{f(y)})] = -\mathbb{E}[\log(f(y))] \\ &= -\int_{-\infty}^{\infty} f(y)\log(f(y))dy,\end{aligned} \tag{3}$$

where $\log(\frac{1}{f(y)})$ is the information content of income.

The following example illustrates how entropy can be used to measure inequality. Consider countries with uniform, gamma, exponential, lognormal, and Pareto income distributions. Figure 1 shows the Lorenz curves for these income distributions. The solid black line represents perfect equality. The deviation from the equality line leads to higher inequality and a greater Gini coefficient. The

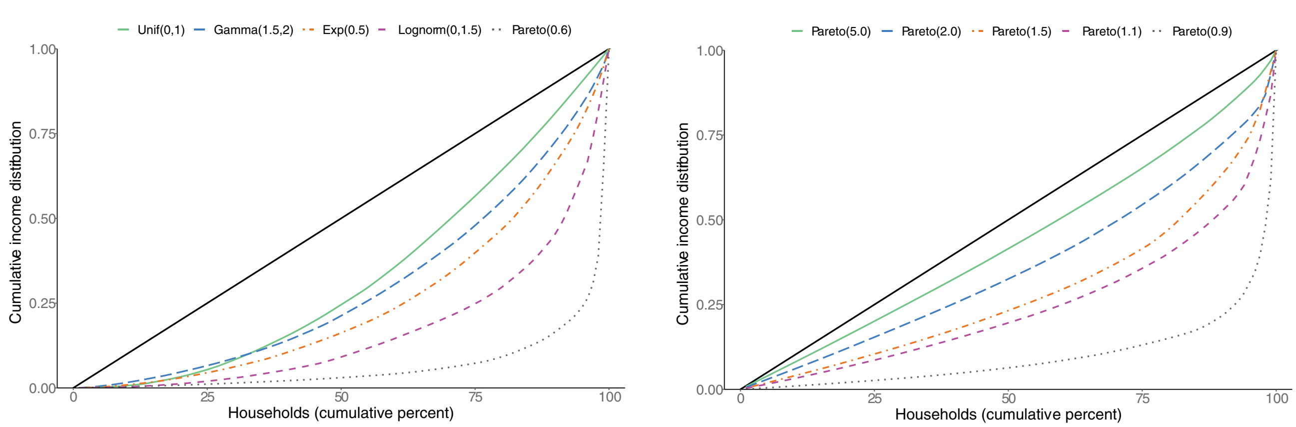

Figure 1. Lorenz curves for countries with different income distributions. The solid black line represents the equality line.

left panel compares countries with different income distributions. Income is equally distributed in a country with a uniform distribution (the green line). Inequality increases for $Lognormal(0, 1.5)$ and $Pareto(.6)$ (the dashed red and dotted grey lines). The Pareto distribution with shape parameter α less than one represents the highest inequality. The right panel illustrates the Pareto densities with a wider range of α. As the shape parameter of Pareto declines, the Lorenz curves deviate from the equality line. Table 1 provides Gini and entropy measures for the distributions shown in Figure 1. We expect a higher Gini for higher inequality. Entropy also increases as inequality rises. For $Pareto(.6)$ and $Pareto(.9)$, Gini coefficients equal one and entropies are the highest.

$\Gamma(\cdot)$, $\psi(\cdot)$, and γ are the gamma, digamma, and incomplete gamma functions. Φ is the CDF of a standard normal, α and β are shape and scale parameters.

Other measures use the notion of entropy to construct inequality measures. See Theil (1967) for example. Table 2 provides the inequality measures for the uniform, gamma, exponential, lognormal, and Pareto densities. To compare the degree of inequality between income levels, we can employ information-theoretic divergence tools that measure the distance between two income distributions (Ebert 1984; Magdalou and Nock 2011). One helpful tool for comparing income distributions is the Kullback–Leibler (KL) divergence. The KL quantifies the proximity of probability distributions (Kullback and Leibler 1951). This divergence measure is also known as relative entropy and can be interpreted as the information gain achieved when $f_2(y)$ is used instead of $f_1(y)$ (Soofi 1994; Soofi and Retzer 2002). The KL measures the divergence between two income distributions $f_1(y)$ and $f_2(y)$ and can be written as

$$\mathcal{D}_{KL}(f_1, f_2) = \mathbb{E}_f(\log\frac{f_1(y)}{f_2(y)}) = \int_{-\infty}^{\infty} f_1(y)\log\frac{f_1(y)}{f_2(y)}dy. \quad (4)$$

Using Jensen's inequality, we can show that $\mathcal{D}_{KL}(f_1, f_2) \geq 0$. Also, $\mathcal{D}_{KL}(f_1, f_2)$ is convex, and $\mathcal{D}_{KL}(f_1, f_2) = 0$ if and only if $f_1(y) = f_2(y)$ (Kullback 1959). The Kullback–Leibler divergence measure, however, is asymmetric. That is, $\mathcal{D}_{KL}(f_1, f_2) \neq \mathcal{D}_{KL}(f_2, f_1)$. This asymmetry property states that the distance from f_1 to f_2 is different than the distance from f_2 to f_1 and can be problematic in the inequality context, resulting in different values of inequality for the same income distribution. The symmetric measure of divergence, known as $\mathcal{J}$-divergence (Jeffreys 1946, 1948), can be used instead. $\mathcal{J}$-divergence is measured by the average of two KL distances between two probability distributions and is given by

$$\mathcal{D}_J(f_1, f_2) = \frac{\mathcal{D}_{KL}(f_1, f_2) + \mathcal{D}_{KL}(f_2, f_1)}{2}. \quad (5)$$

KL and J-divergence represent inequality by comparing income distribution to the uniform density. Moving away from the uniform income distribution indicates greater inequality. Recently, Rohde (2016) uses the J-divergence to capture inequality, highlighting the properties of this divergence measure. The following example demonstrates how the divergence measures can be applied for inequality comparison. Consider five hypothetical countries with individuals' income levels available from household survey data. From the surveys, we can compute income distribution for the i-th country f_i, $i = 1, \cdots, 5$ as the proportion of the population in low-, middle-, and high-income levels. For example, income is equally distributed in Country 1. Table 3 shows the income distributions for the

Table 1. Gini and entropy measures of known income distributions.

Distribution	Gini	Entropy	Distribution	Gini	Entropy
$Uniform(0, 1)$	0.33	0.00	$Pareto(5.0)$	0.11	0.02
$Gamma(1.5, 2)$	0.42	0.67	$Pareto(2.0)$	0.33	0.30
$Exponential(0.5)$	0.50	1.69	$Pareto(1.5)$	0.50	0.90
$Lognormal(0, 1.5)$	0.71	1.82	$Pareto(1.1)$	0.83	1.81
$Pareto(0.6)$	1.00	3.17	$Pareto(0.9)$	1.00	2.21

Table 2. Inequality measures of known income distributions.

Income Distribution	PDF	Gini	Entropy
Uniform	$\frac{1}{b-a}$	$\frac{b-a}{3(b+a)}$	$\log(b-a)$
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}y^{\alpha-1}e^{-\beta y}$	$\frac{\Gamma\left(\frac{2\alpha+1}{2}\right)}{\alpha\Gamma(\alpha)\sqrt{\pi}}$	$\alpha - \log\beta + \log\Gamma(\alpha) + (1-\alpha)\psi(\alpha)$
Exponential	$\frac{1}{\beta}e^{-\frac{1}{\beta}y}$	1/2	$1 + \log\beta$
Lognormal	$\frac{1}{y\sigma\sqrt{2\pi}}e^{-\frac{\log y^2}{2\sigma^2}}$	$2\Phi\left(\frac{\sigma}{\sqrt{2}}\right) - 1$	$\log(\sigma\sqrt{2\pi e})$
Pareto	$\frac{\alpha}{y^{1+\alpha}}$	$\begin{cases} 1 & 0<\alpha<1 \\ \frac{1}{2\alpha-1} & \alpha \geq 1 \end{cases}$	$1 + \frac{1}{\alpha} - \log\alpha$

Table 3. Income distributions comparison.

Income level	f_1	f_2	f_3	f_4	f_5
Low	0.33	0.15	0.35	0.05	0.35
Middle	0.33	0.65	0.45	0.60	0.60
High	0.33	0.20	0.20	0.35	0.05
Entropy	1.10	0.89	1.05	0.82	0.82
$\mathcal{D}_{KL}(f_1, f_j)$	0.00	0.20	0.04	0.41	0.41
$\mathcal{D}_J(f_1, f_j)$	0.00	−0.21	−0.05	−0.35	−0.35

$\mathcal{D}_{KL}(f_1, f_j)$ and $\mathcal{D}_J(f_1, f_j)$ denote the KL and J-divergence, and f_1 is the reference distribution.

three income levels. Entropies and the two divergence measures are reported. For $\mathcal{D}_{KL}(f_1, f_j)$ and $\mathcal{D}_J(f_1, f_j)$, f_1 (uniform density) is considered as the reference distribution. For f_1, entropy is highest, and the divergence measures are zero since f_1 is the reference distribution. Although the probabilities of the high-income level are equal in Countries 2 and 3, the entropy for Country 3 is higher than for Country 2 because the probabilities of low and middle-income levels are closer in Country 3. Also, KL and the absolute value of J-divergence are lower in Country 3, signalling lower inequality. This example illustrates the usefulness of divergence measures in capturing and comparing inequality.

III. Income inequality using the world values survey

This section employs household survey data to compute and compare income inequality measures among nations. We use the World Values Survey (WVS) data. The WVS is an international survey program completed in seven waves for the period 1981–2020: waves 1 (1981–1983); 2 (1990–1992); 3 (1995–1998); 4 (2000–2004); 5 (2005–2008); 6 (2010–2014), and 7 (2017–2020). The survey questions and the number of countries have changed over time. For consistency, we consider Wave 7 released by Haerpfer et al. (2020). Table 4 provides the list of countries and the sample size (number of participants in the survey). The WVS provides income data (i.e. wages, salaries, pensions, and other forms of income) into country-specific income deciles. Specifically, WVS respondents' income levels are recorded as low-income (groups 1–3), middle-income (groups 4–7), and high-income (groups 8–10). 'Do not know', 'no answer', 'not asked', and 'missing; unknown' responses are removed from the data set, remaining 82,776 observations for the 57 countries.

We first compute the proportions of the low-, middle-, and high-income populations. We then construct the inequality measures and provide the corresponding rankings. Table 5 presents the income distributions, Gini index, entropy, divergence measures, and the corresponding rankings.

Table 4. Countries included in the WVS-7 (2017–2020) dataset.

No	Countries	Sample	No	Countries	Sample	No	Countries	Sample
1	Andorra	1004	21	Iran	1499	41	Philippines	1200
2	Argentina	1003	22	Iraq	1200	42	Puerto Rico	1127
3	Australia	1813	23	Japan	1353	43	Romania	1257
4	Armenia	1223	24	Jordan	1203	44	Russia	1810
5	Bangladesh	1200	25	Kazakhstan	1276	45	Serbia	1046
6	Bolivia	2067	26	Kenya	1266	46	Singapore	2012
7	Brazil	1762	27	Kyrgystan	1200	47	South Korea	1245
8	Canada	4018	28	Lebanon	1200	48	Taiwan	1223
9	Chile	1000	29	Libya	1196	49	Tajikistan	1200
10	China	3036	30	Macau	1023	50	Thailand	1500
11	Colombia	1520	31	Malaysia	1313	51	Tunisia	1208
12	Cyprus	1000	32	Mexico	1739	52	Turkey	2415
13	Ecuador	1200	33	Mongolia	1638	53	Ukraine	1289
14	Egypt	1200	34	Morocco	1200	54	United States	2596
15	Ethiopia	1230	35	Myanmar	1200	55	Venezuela	1190
16	Germany	1528	36	New Zealand	1057	56	Vietnam	1200
17	Greece	1200	37	Nicaragua	1200	57	Zimbabwe	1215
18	Guatemala	1203	38	Nigeria	1237			
19	Hong Kong	2075	39	Pakistan	1995			
20	Indonesia	3200	40	Peru	1400			

Table 5. Probabilities of households in different income levels, inequality, and the ranks.

Country	Low	Middle	High	$\mathcal{G}$	$\mathcal{H}$	$\mathcal{D}_{KL}$	$\mathcal{D}_J$	Rank$_{\mathcal{G}}$	Rank$H$	Rank$_{\mathcal{D}_{KL}}$	Rank$_{\mathcal{D}_J}$
Andorra	0.15	0.75	0.10	0.41	0.73	0.38	−0.38	12	14	36	17
Argentina	0.16	0.77	0.07	0.38	0.68	0.47	−0.45	9	9	43	12
Australia	0.25	0.68	0.07	0.47	0.79	0.36	−0.34	22	23	34	23
Armenia	0.23	0.65	0.11	0.51	0.86	0.26	−0.25	34	37	13	44
Bangladesh	0.17	0.66	0.17	0.51	0.88	0.21	−0.22	35	46	7	47
Bolivia	0.22	0.66	0.12	0.5	0.86	0.24	−0.24	30	38	11	45
Brazil	0.41	0.54	0.06	0.54	0.87	0.33	−0.29	45	42	28	32
Canada	0.15	0.73	0.12	0.43	0.77	0.33	−0.34	15	18	29	24
Chile	0.22	0.72	0.05	0.43	0.72	0.50	−0.44	16	13	45	13
China	0.37	0.60	0.03	0.50	0.78	0.56	−0.44	31	20	51	14
Colombia	0.38	0.50	0.12	0.59	0.97	0.15	−0.15	55	55	3	55
Cyprus	0.15	0.76	0.09	0.39	0.71	0.41	−0.41	11	11	39	16
Ecuador	0.17	0.77	0.06	0.37	0.67	0.5	−0.47	8	8	46	10
Egypt	0.28	0.60	0.12	0.55	0.92	0.19	−0.19	48	50	5	52
Ethiopia	0.15	0.83	0.02	0.29	0.52	0.88	−0.74	1	1	57	1
Germany	0.32	0.60	0.08	0.53	0.87	0.28	−0.26	42	43	17	42
Greece	0.29	0.67	0.04	0.47	0.76	0.51	−0.43	23	17	48	15
Guatemala	0.10	0.66	0.23	0.50	0.84	0.28	−0.27	32	33	18	38
Hong Kong	0.24	0.72	0.04	0.42	0.71	0.54	−0.47	14	12	50	11
Indonesia	0.40	0.52	0.09	0.56	0.92	0.22	−0.21	51	51	9	50
Iran	0.42	0.52	0.06	0.55	0.87	0.33	−0.28	49	44	30	34
Iraq	0.30	0.62	0.08	0.52	0.86	0.29	−0.27	39	39	21	39
Japan	0.40	0.56	0.04	0.52	0.82	0.46	−0.37	40	29	42	20
Jordan	0.48	0.36	0.15	0.62	1.00	0.11	−0.10	56	56	2	56
Kazakhstan	0.10	0.80	0.10	0.34	0.64	0.50	−0.48	6	6	47	6
Kenya	0.27	0.66	0.07	0.49	0.81	0.35	−0.32	29	27	32	25
Kyrgystan	0.19	0.69	0.12	0.47	0.83	0.27	−0.28	24	31	15	35
Lebanon	0.16	0.83	0.02	0.29	0.53	0.86	−0.73	2	2	56	2
Libya	0.16	0.70	0.14	0.46	0.82	0.27	−0.28	21	30	16	36
Macau	0.16	0.71	0.13	0.45	0.80	0.29	−0.30	19	25	22	31
Malaysia	0.15	0.81	0.04	0.32	0.58	0.66	−0.59	4	3	55	3
Mexico	0.42	0.48	0.10	0.58	0.95	0.19	−0.18	54	54	6	53
Mongolia	0.27	0.64	0.09	0.51	0.86	0.28	−0.26	36	40	19	43
Morocco	0.15	0.72	0.13	0.44	0.79	0.31	−0.32	17	24	23	26
Myanmar	0.13	0.80	0.08	0.34	0.65	0.48	−0.48	7	7	44	7
New Zealand	0.32	0.57	0.10	0.56	0.92	0.22	−0.2	52	52	10	51
Nicaragua	0.35	0.58	0.07	0.54	0.87	0.31	−0.27	46	45	24	40
Nigeria	0.32	0.56	0.12	0.57	0.94	0.17	−0.17	53	53	4	54
Pakistan	0.28	0.39	0.33	0.66	1.09	0.00	−0.01	57	57	1	57
Peru	0.34	0.57	0.09	0.55	0.90	0.24	−0.22	50	49	12	48
Philippines	0.23	0.68	0.08	0.48	0.80	0.35	−0.32	27	26	33	27
Puerto Rico	0.33	0.60	0.07	0.53	0.86	0.31	−0.28	43	41	25	37
Romania	0.21	0.65	0.14	0.51	0.88	0.21	−0.22	37	47	8	49
Russia	0.16	0.72	0.12	0.44	0.78	0.32	−0.32	18	21	26	28
Serbia	0.27	0.64	0.08	0.51	0.84	0.32	−0.29	38	34	27	33
Singapore	0.20	0.76	0.05	0.38	0.68	0.51	−0.48	10	10	49	8
South Korea	0.26	0.65	0.09	0.50	0.85	0.28	−0.27	33	36	20	41
Taiwan	0.24	0.68	0.08	0.47	0.81	0.33	−0.32	25	28	31	29
Tajikistan	0.07	0.82	0.11	0.31	0.59	0.57	−0.55	3	4	52	5
Thailand	0.30	0.61	0.09	0.53	0.88	0.26	−0.24	44	48	14	46
Tunisia	0.15	0.75	0.10	0.41	0.73	0.38	−0.38	13	15	37	18
Turkey	0.29	0.67	0.03	0.47	0.73	0.60	−0.48	26	16	53	9
Ukraine	0.29	0.66	0.05	0.48	0.78	0.44	−0.38	28	22	40	19
United States	0.20	0.71	0.08	0.45	0.77	0.38	−0.36	20	19	38	21
Venezuela	0.33	0.61	0.06	0.52	0.84	0.36	−0.32	41	35	35	30
Vietnam	0.14	0.81	0.05	0.32	0.60	0.61	−0.56	5	5	54	4
Zimbabwe	0.51	0.45	0.04	0.54	0.83	0.45	−0.36	47	32	41	22

Low, middle, and high provide proportions of the low-, middle-, and high-income populations. $\mathcal{G}$, $\mathcal{H}$, $\mathcal{D}_{KL}$, and $\mathcal{D}_J$ denote Gini, entropy, Kullback-Leibler, and *J*-divergence. The uniform distribution is the benchmark for computing the divergence measures.

The uniform distribution is the benchmark for computing the divergence measures. The countries are listed alphabetically. Pakistan has 33% (highest proportion) of people in the high-income level, whereas Ethiopia and Lebanon have 2% (lowest proportion). Zimbabwe and Tajikistan hold the highest and lowest percentage of people at the low-income level, respectively. Ethiopia is ranked first based on the Gini, entropy, and *J*-divergence measures, and Pakistan is ranked last. Figure 2 summarizes these results and presents the rankings based on Gini, entropy, and *J*-divergence. The divergence representation highlights the evenness across income levels, while the Gini coefficient

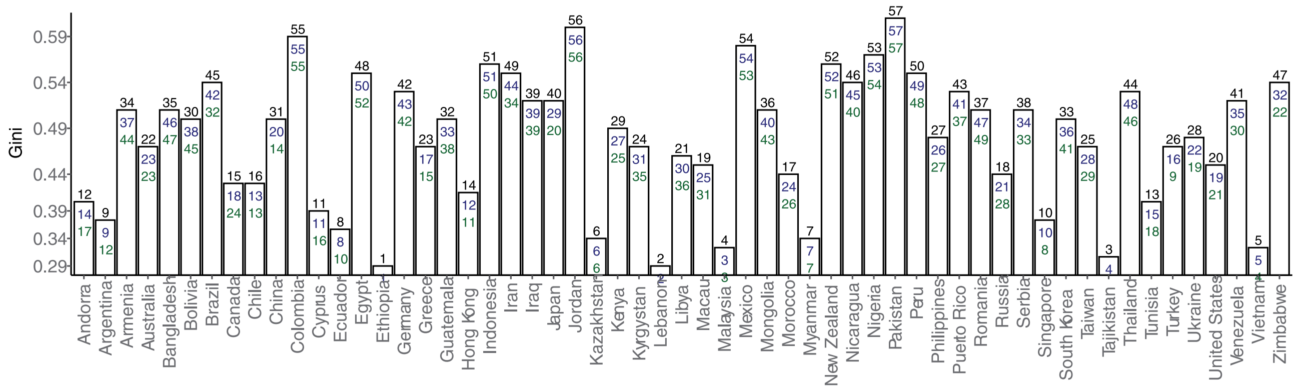

Figure 2. Countries with corresponding Gini (black), entropy (blue), and *J*-divergence (green) ranks.

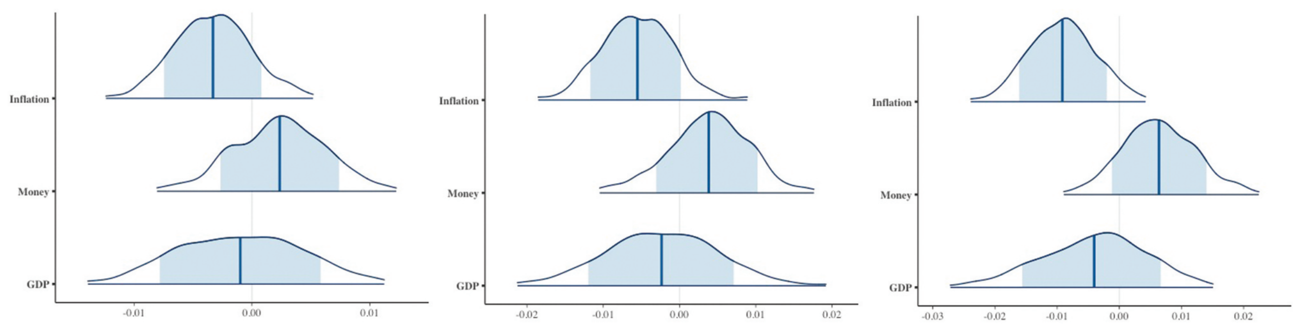

Figure 3. Posterior distributions of Gini (left), entropy (middle), and *J*-divergence (right) models, 80% intervals (shaded).

represents a proportion of low-income individuals. Lower values for the *J*-divergence represent a more uneven distribution across income levels. Higher Gini coefficients indicate greater inequality, with low-income individuals receiving smaller percentages of the total income. For example, Ethiopia has a lower Gini coefficient than the United States because it has a smaller proportion of low-income individuals. However, Ethiopia has a smaller value of *J*-divergence than the United States because income distribution is more uneven across income levels. The Gini with the J-divergence measure provides insight into the proportion of low-income individuals while accounting for the discrepancy among income levels.

IV. The effects of macroeconomic outcomes on income inequality

The theoretical and empirical literature suggests the relationship between macroeconomic outcomes and income inequality. Theoretical works explore this relationship through endogenous growth and general dynamic equilibrium models. Alesina and Rodrik (1994), for example, theoretically show a negative relationship between inequality and growth. They argue that more equitable income distribution lowers the equilibrium level of capital taxation, leading to higher growth. Heer and Sussmuth (2007) examine the effects of inflation on the wealth distribution in a general equilibrium model, suggesting high inflation increases inequality. On the other hand, studies empirically examine the correlation between macroeconomic outcomes and inequality. For example, the relationship between inequality and economic growth (Deininger and Squire 1996; Li and Zou 1998), trade (Huang et al. 2022), monetary growth (Bagchi, Curran, and Fagerstrom 2019), and inflation (Doepke and Schneider 2006) are documented.

This section examines the relationship between macroeconomic variables and inequality using the measures computed in the previous section. We apply a Bayesian framework and consider the main macroeconomic covariates: inflation, broad money growth, and GDP per capita growth. The data are obtained from the World Bank World Development Indicators. For consistency, we utilize the average observations from 2017 to 2020 to match the data frequency in WVS-Wave 7 (2017–2020). Andorra, Taiwan, and Venezuela are removed from the analysis due to data unavailability. Consider the following linear regression model

$$y \sim \mathcal{N}(\mu, \sigma^2), \quad (6)$$

where y is the inequality measure, and $\mu = \mathbf{X}\beta$. We then combine the information from the data with the prior and construct the posterior distributions. Let $\theta = (\beta, \sigma)$ be the vector of parameters. Assuming the regression parameters are independent, the joint prior PDF is the product of the marginal PDFs $f(\theta)$ defined by

$$f(\theta) = \prod_{i=1}^{n} f(\theta_i). \quad (7)$$

The likelihood function is given by the joint PDF of y, which is the product of the marginal PDFs. That is,

$$\mathcal{L}(\theta|y) = \prod_{i=1}^{n} f(y_i|\theta). \quad (8)$$

Thus, the posterior is given by

$$f(\theta|y) \propto \text{Prior PDF} \times \text{Likelihood Function} = \prod_{i=1}^{n} f(\theta_i) \prod_{i=1}^{n} f(y_i|\theta). \quad (9)$$

The merits of this Bayesian framework are two-fold: (1) it incorporates a prior (or assumed knowledge) and then updates the beliefs using the evidence, and (2) it takes into account uncertainty by incorporating predictive distributions, which reduces the likelihood of overfitting. See Zellner (1996) for details. The Monte Carlo Markov Chain (MCMC) simulation structure is as follows. The number of iterations for each Markov chain is 10,000 with thinning of 5 observations. We use a Gibbs sampler to draw samples from posterior distributions. The results of the Bayesian models for the inequality measures are summarized in Table 6. The median and mean estimates are computed from the MCMC simulation. The MAD gives the median absolute deviation. Figures 3 and 4 provide the posterior density plots with 80% intervals and MCMC scatterplots of the Bayesian models.

Table 6. Bayesian normal regression models.

Parameter	Median	MAD	Mean	SD	0.95% CI
Gini					
Intercept	0.473	0.033	0.474	0.034	[0.406, 0.538]
Inflation	−0.003	0.003	−0.003	0.003	[−0.009, 0.003]
Money growth	0.002	0.004	0.002	0.004	[−0.006, 0.010]
GDP growth	−0.001	0.006	−0.001	0.005	[−0.011, 0.009]
Entropy					
Intercept	0.802	0.045	0.803	0.045	[0.705, 0.886]
Inflation	−0.006	0.005	−0.006	0.005	[−0.015, 0.003]
Money growth	0.004	0.005	0.004	0.005	[−0.007, 0.014]
GDP growth	−0.002	0.008	−0.002	0.007	[−0.016, 0.013]
J-divergence					
Intercept	−0.341	0.050	−0.340	0.052	[−0.436, −0.236]
Inflation	−0.009	0.005	−0.009	0.005	[−0.019, 0.002]
Money growth	0.006	0.006	0.006	0.006	[−0.004, 0.020]
GDP growth	−0.004	0.009	−0.004	0.009	[−0.020, 0.013]

The median and mean estimates are computed from the MCMC simulation. The MAD gives the median absolute deviation. The number of iterations for each Markov chain is 10,000 with thinning of 5 observations.

Our findings suggest a positive relationship between money growth and inequality. This result is consistent with the findings of Bagchi, Curran, and Fagerstrom (2019). The expansionary monetary policy provides excess money to the banks. This new money is usually funnelled through wealthy investments leading to higher inequality (Doepke and Schneider 2006; Meh, Ríos-Rull, and Terajima 2010). We also find a negative relationship between inflation and inequality. Jin (2009) studies this relationship through a monetary endogenous growth model. She shows that higher money growth increases inflation and its effect on income inequality depends on the size of capital and skill heterogeneity. The negative relationship between inflation and inequality is expected when capital heterogeneity dominates skill heterogeneity. The literature has reported different results on the relationship between growth and inequality (Barro 2000; Ivaschenko 2002; Lundberg and Squire 2003). We find a negative relationship between GDP growth and income inequality, consistent with the theoretical results of Alesina and Rodrik (1994).

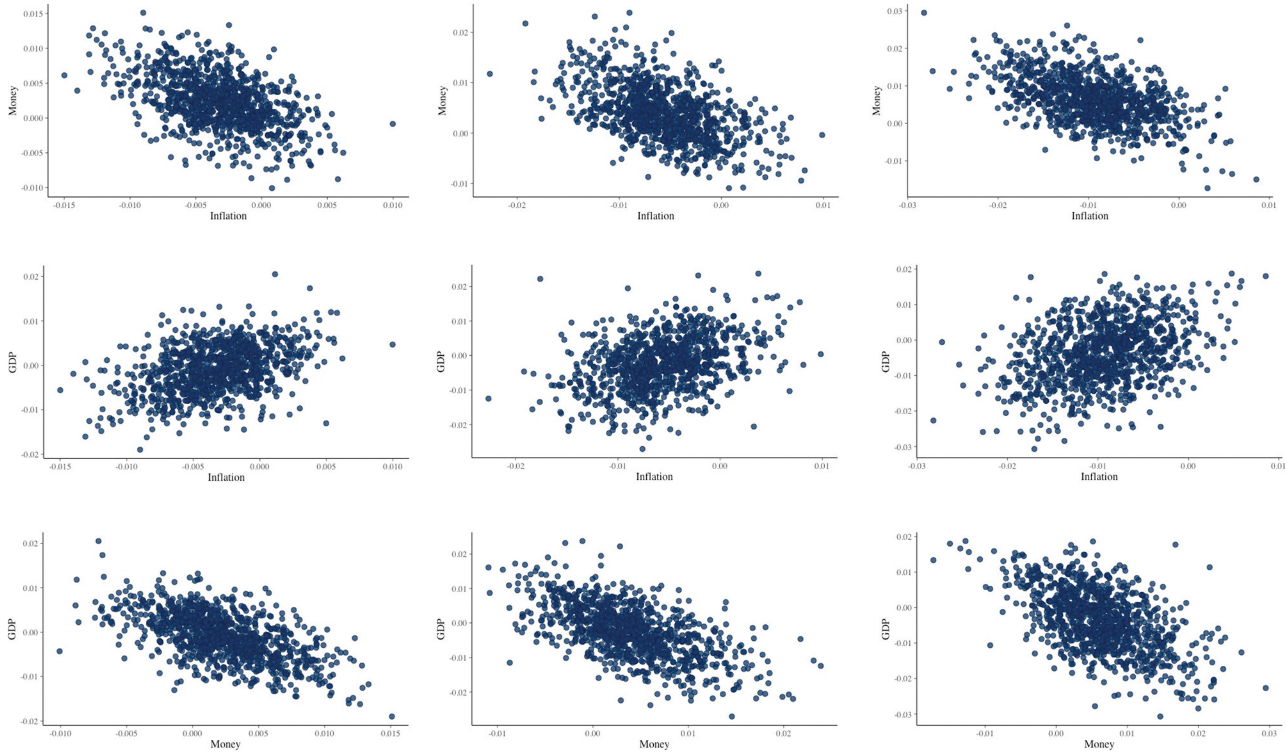

Figure 4. MCMC scatterplots of Gini (left), entropy (middle), and *J*-divergence (right) models.

V. Conclusions

The Gini index and the information-theoretic measures provide a meaningful understanding of income inequality. In this paper, we compare widely used inequality indices and discuss divergence measures for comparing the degree of inequality between income levels among countries. Specifically, we use the World Values Survey data, including 57 countries with low, middle, and high household income levels, to estimate the Gini index, entropy, Kullback-Leibler, and *J*-divergence measures. Considering the J-divergence measure along with the Gini index provides a fuller picture of a country's degree of inequality. Higher Gini values indicate greater inequality, with low-income individuals receiving smaller percentages of the total income. On the other hand, lower values for the J-divergence represent less even income distributions across income levels. Examining the effects of macroeconomic outcomes on inequality measures suggests that the effects of inflation, money growth, and GDP growth on inequality are more economically significant when the J-divergence measure is used. Results show that inflation and GDP growth negatively affect inequality. However, money growth positively impacts inequality. Our findings suggest that macroeconomic outcomes can also be employed to capture income inequality.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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